Parametric resonance and stochastic stability of a vibro-impact system under bounded noise excitation

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Abstract The vibro-impact system has more complex and variable dynamic behavior, and its research has attracted the attention of many scholars. However, the effect of resonance on the dynamic characteristics of vibro-impact systems is rarely explored. In this paper, the parametric resonance and stochastic stability of a vibro-impact system excited by bounded noise parameters are investigated. For weak noise excitations, the approximate analytical results of the moment Lyapunov exponent and the largest Lyapunov exponent are calculated by using the method of singular perturbation and Fourier series expansion, which are in good agreement with those obtained by Monte Carlo numerical simulation. Then, based on the moment Lyapunov exponent and the largest Lyapunov exponent, the effects of parametric resonance and different parameters on the stochastic stability of the vibro-impact system are investigated in detail.

Keywords *Parametric resonance; Stochastic stability*; *Moment Lyapunov exponent*; *Vibro-impact system.*

1. Introduction

Vibro-impact phenomena are commonly observed in mechanical and engineering structures, such as vibratory pile drivers, piezoelectric energy harvesting devices, and spacecraft docking. The analysis of such systems is complicated by the fact that collisions cause transient changes in the system elements, resulting in strong nonlinearity and

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non-smoothness. Additionally, random factors often accompany the vibro-impact system. The combination of strong nonlinearity, non-smoothness, and random factors leads to a more complex and diverse dynamic behavior of a dynamical system. Therefore, the study of the dynamic characteristics of vibro-impact systems under random disturbances has attracted the attention of numerous scholars[1-6].

In recent decades, scholars in mathematics, mechanics, engineering, and other related fields have made significant achievements in studying the stochastic dynamic behavior of vibro-impact systems[7-9]. In 1972, Nayak[10] conducted significant research on the stochastic dynamic characteristics of vibro-impact systems. Huang[11] applied the stochastic averaging method to analyze the stationary response of a random vibro-impact system. Based on the average energy loss, Gu[12] proposed a new method for studying the response problem of random vibro-impact systems. The feasibility of the method was verified using a numerical example. Wang[13] proposed a new path integral algorithm for calculating the system response by introducing the concept of an absorption surface and an impact completion condition for random vibro-impact systems. For the vibro-impact system with the interaction of random disturbances and external deterministic excitation, Leng[14] analyzed the stochastic stability of the system by considering the activation energy of the attractor. Ma[15] proposed a method to solve the response of random vibro-impact systems without introducing nonsmooth transformations. The study of the stochastic bifurcation of vibro-impact systems has also sparked the interest of numerous scholars and has yielded significant results[16-20].

Stability and bifurcation behavior have been a hot topic in the field of nonlinear dynamics. The largest Lyapunov exponent and moment Lyapunov exponent are important indices for describing the stochastic stability and bifurcation behavior of stochastic dynamical systems. The *p*th moment Lyapunov exponent is presented as

$$\Lambda\left(p,\mathbf{x}_0\right)=\lim_{t\to+\infty}\frac{1}{t}\log E\left[\left\|\mathbf{x}(t;\mathbf{x}_0)\right\|^p\right], \tag{1}$$

where $\mathbf{x}\left(t;\mathbf{x}_0\right)$ is the stationary solution for a random dynamical system; $\mathbf{x}_0$ is the initial value; $E\left[\bullet\right]$ is the expected value. The random dynamical system is *p*th moment stable as $\Lambda\left(p,\mathbf{x}_0\right)<0$. Arnold[21] pointed out that under certain conditions, the limits of Eq. (1) is independent of the initial value $\mathbf{x}_0$. And the largest Lyapunov exponent can be expressed as[22]

$$\lambda=\frac{\mathrm{d}}{\mathrm{d}p}\Lambda\left(p\right)\bigg|_{p=0}=\lim_{t\to\infty}\frac{1}{t}\log\left\|\mathbf{x}\left(t,\mathbf{x}_0\right)\right\|. \tag{2}$$

In recent years, research on the *p*th moment Lyapunov exponent of random dynamical systems has achieved numerous satisfactory results[23-28]. However, the study of random vibro-impact systems is still in its early stages and requires further development. Recently, Hu[29] calculated the *p*th moment Lyapunov exponent of a vibro-impact system excited by white Gaussian noise, and investigated the system's stochastic stability.

In practical applications, the random excitation is often characterized by a Gaussian white noise process, a real noise process, or a bounded noise process. The Gaussian white noise is an idealized model, while the real noise process follows the normal distribution and is unbounded, which is not applicable in many practical engineering applications. The bounded noise is a type of realistic and general random disturbance model, which is widely used in practical engineering modeling[30-34]. Moreover, a dynamical system with bounded noise disturbance will exhibit more complex dynamic behavior, such as resonance. The occurrence of resonance can have either a positive or negative impact on the stability of the system[35-39]. For a two-degree-of-freedom system excited by bounded noise, Zhu[40] studied the phenomenon of parametric resonance in the system and discussed its influence on the stochastic stability of the system. Deng[41] modeled the random excitation as a bounded noise process and analyzed the stability of the viscoelastic plate under various resonance scenarios by calculating the pth moment Lyapunov exponent and largest Lyapunov exponent. However, the impact of bounded noise and resonance on the stochastic stability of vibro-impact systems still needs to be studied. In order to address this gap, we will examine the phenomenon of parametric resonance of a vibro-impact system that is subjected to bounded noise excitation. Furthermore, by determining the pth moment Lyapunov exponent of the system, the influence of parameter resonance and various system parameters on the stability of the vibro-impact system is investigated.

2. Formulation

Consider a Rayleigh-Van der Pol vibro-impact system with a single rigid barrier under bounded noise excitation, whose governing equation is represented as

$$\begin{aligned} &\ddot{x}+\omega^2 x-\left(c_1+\xi\left(t\right)\right)\dot{x}+\left(c_2 x^2+c_3\dot{x}^2\right)\dot{x}=0, \quad x>0 \\ &\dot{x}_+=-r\dot{x}_-, \quad x=0. \end{aligned} \tag{3}$$

where ω represents the natural frequency; c_1 and c_2, c_3 denote the linear and nonlinear damping coefficients, respectively. r is the recovery coefficient , recovery coefficient, which is used to describe the impact energy loss (i.e., $0<r\le 1$). $\dot{x}_-$, $\dot{x}_+$ are the impact velocity and rebound velocity, respectively. $\xi\left(t\right)$ is an ergodic random process with zero mean, which is given as

$$\xi(t) = \cos(\Omega t + \sigma W(t) + \psi), \tag{4}$$

with the power spectral density

$$S(\omega) = \frac{\sigma^2\left(\omega^2 + \Omega^2 + \frac{1}{4}\sigma^4\right)}{2\left((\omega+\Omega)^2 + \frac{1}{4}\sigma^4\right)\left((\omega-\Omega)^2 + \frac{1}{4}\sigma^4\right)},$$

where $W(t)$ is a normal Wiener process; ψ is a random number that follows a uniform distribution in the interval $[0, 2\pi]$; Ω is the frequency of the bounded noise; σ is the noise intensity. By introducing random number ψ, the bounded noise $\xi(t)$ becomes a stationary random process. And $\xi(t)$ is a narrow-band random process when σ is small.

In order to facilitate analytical processing, the vibro-impact system (3) is equivalent to a smooth dynamic system by introducing the non-smooth coordinate transformation. The Zhuravlev transformation [42] is represented as

$$\begin{aligned} x &= |y|, \\ \dot{x} &= \dot{y}\,\mathrm{sgn}(y), \\ \ddot{x} &= \ddot{y}\,\mathrm{sgn}(y). \end{aligned} \tag{5}$$

where

$$\mathrm{sgn}(y) = \begin{cases} 1, & y > 0, \\ 0, & y = 0, \\ -1, & y < 0. \end{cases}$$

Substituting Eq.(5) into Eq. (3) , and with the aid of the Dirac delta function[43, 44], the governing equation is converted to

$$\ddot{y} + \omega^2 y - \varepsilon\left(c_1 - c_2 y^2 - c_3 \dot{y}^2\right)\dot{y} + \varepsilon(1-r)\dot{y}|\dot{y}|\delta(y) - \varepsilon\xi(t)\dot{y} = 0. \tag{6}$$

where the random term, impact energy loss, and damping term are assumed to be a small quantity of ε ($0 < \varepsilon \ll 1$ is a small parameter); $\delta(\bullet)$ is a Dirac delta function. And the random process is considered. i.e.,

$$\begin{aligned} \xi(t) &= \cos\eta(t), \\ \eta(t) &= \Omega t + \varepsilon^{1/2}\sigma W(t). \end{aligned} \tag{7}$$

Neglecting the nonlinear terms, and applying the transformation $y = y_1, \dot{y} = \omega y_2$, Eq. (6) becomes

$$\begin{aligned} \dot{y}_1 &= \omega y_2, \\ \dot{y}_2 &= -\omega y_1 + \varepsilon\left(c_1 + (r-1)\omega|y_2|\delta(y_1) + \xi(t)\right)y_2. \end{aligned} \tag{8}$$

Letting $y_1 = e^{\rho}\cos\theta, y_2 = -e^{\rho}\sin\theta, \theta \in [0,\pi]$, Eq. (8) is changed to

$$\begin{aligned} \mathrm{d}\rho &= \varepsilon\left(q_2(\theta) + q_1(\theta)\cos\eta\right)\mathrm{d}t, \\ \mathrm{d}\theta &= \left(\omega + \varepsilon\left(s_2(\theta) + s_1(\theta)\cos\eta\right)\right)\mathrm{d}t, \end{aligned} \tag{9}$$

where

$$q_1(\theta) = \frac{1}{2}(1-\cos 2\theta), \qquad s_1(\theta) = \frac{1}{2}\sin 2\theta,$$

$$q_2(\theta) = \frac{1}{2}\left(c_1 + (r-1)\omega|\sin\theta|\delta(\cos\theta)\right)(1-\cos 2\theta),$$

$$s_2(\theta) = \frac{1}{2}\left(c_1 + (r-1)\omega|\sin\theta|\delta(\cos\theta)\right)\sin 2\theta.$$

3. Moment Lyapunov exponent

Let $\Omega = \Omega_0 + \varepsilon\Delta, \Omega_0 = \alpha\omega, \eta = z + \alpha\theta$, where α is a constant, Δ denotes the detuning parameter, and Ω_0 is the central frequency. According to Eqs. (7) and (9), one easily finds that the random processes (θ,η) are independent of the variable ρ. Thus, the processes (θ, z) alone form a diffusive Markov process with the following generator:

$$\begin{aligned} L_\varepsilon(p) = \omega\frac{\partial}{\partial\theta} &+ \varepsilon\left(\frac{\sigma^2}{2}\frac{\partial^2}{\partial z^2} + \left(\Delta - \alpha\left(s_2(\theta) + s_1(\theta)\cos(z+\alpha\theta)\right)\right)\frac{\partial}{\partial z}\right) \\ &+ \varepsilon\left(\left(s_2(\theta) + s_1(\theta)\cos(z+\alpha\theta)\right)\frac{\partial}{\partial\theta} + p\left(q_2(\theta) + q_1(\theta)\cos(z+\alpha\theta)\right)\right). \end{aligned} \tag{10}$$

i.e.,

$$L_\varepsilon(p) = L_0(p) + \varepsilon L_1(p), \tag{11}$$

where

$$L_0(p) = \omega\frac{\partial}{\partial\theta},$$

$$\begin{aligned} L_1(p) = \frac{\sigma^2}{2}\frac{\partial^2}{\partial z^2} &+ \left(\Delta - \alpha\left(s_2(\theta) + s_1(\theta)\cos(z+\alpha\theta)\right)\right)\frac{\partial}{\partial z} \\ &+ \left(s_2(\theta) + s_1(\theta)\cos(z+\alpha\theta)\right)\frac{\partial}{\partial\theta} + p\left(q_2(\theta) + q_1(\theta)\cos(z+\alpha\theta)\right). \end{aligned}$$

According to Arnold [21] and Khasminskii [45], the *p*th moment Lyapunov exponent $\Lambda(p)$ of system (9) is the principal simple eigenvalue of the operator $L_\varepsilon(p)$, i.e.,

$$L_\varepsilon(p)T(p) = \Lambda(p)T(p). \tag{12}$$

Based on the results of Khasminskii [45], both $\Lambda(p)$ and $T(p)$ are expanded in the power series of ε, respectively, i.e.,

$$\begin{aligned}\Lambda(p)&=\Lambda_0(p)+\varepsilon\Lambda_1(p)+\varepsilon^2\Lambda_2(p)+\cdots+\varepsilon^n\Lambda_n(p)+\cdots,\\ T(p)&=T_0(p)+\varepsilon T_1(p)+\varepsilon^2T_2(p)+\cdots+\varepsilon^nT_n(p)+\cdots.\end{aligned}\tag{13}$$

Substituting Eqs. (11) and (13) into Eq. (12), one easily obtains

$$\varepsilon^0:\left(L_0(p)-\Lambda_0(p)\right)T_0(p)=0,\tag{14}$$

$$\varepsilon^1:\left(L_0(p)-\Lambda_0(p)\right)T_1(p)=\left(\Lambda_1(p)-L_1(p)\right)T_0(p),\tag{15}$$

$$\begin{aligned}\varepsilon^2&:\left(L_0(p)-\Lambda_0(p)\right)T_2(p)=\left(\Lambda_1(p)-L_1(p)\right)T_1(p)+\left(\Lambda_2(p)-L_2(p)\right)T_0(p),\\ &\;\;\vdots\end{aligned}\tag{16}$$

3.1 Zeroth-order perturbation

From the definition of pth moment Lyapunov exponent $\Lambda(p)$, it is clear that $\Lambda_0(p)\equiv 0$ for any p. Then, Eq. (14) becomes

$$\omega\frac{\partial T_0(p)}{\partial\theta}=0.\tag{17}$$

Making use of the separation of variables method and letting $T_0(p)=Z_0(z)\Theta_0(\theta)$ leads to

$$\frac{\dot{\Theta}_0}{\Theta_0}=0.\tag{18}$$

With the help of the periodic boundary condition $\Theta_0(\theta+2\pi)=\Theta_0(\theta)$, we get that $\Theta_0(\theta)$ is a constant. Hence, $T_0(p)=T_0(z,\theta)=T_0(z)$.

The adjoint equation of Eq. (17) is given as

$$L_0^*(p)T_0^*(p)=0.\tag{19}$$

Based on the separation of variables method ,it is easily obtained that

$$T_0^*(p)=T_0^*(z,\theta)=T_0^*(z).\tag{20}$$

3.2 First-order perturbation

According to what we got above, Eq. (15) becomes

$$L_0(p)T_1(p)=\left(\Lambda_1(p)-L_1(p)\right)T_0(p).\tag{21}$$

The solvability condition to Eq. (21) is

$$\begin{aligned}&\left\langle\left(\Lambda_1(p)-L_1(p)\right)T_0(p),\ T_0^*(p)\right\rangle\\&=\int_{-\infty}^{+\infty}T_0^*(z)\int_0^{2\pi}\left(\left(\Lambda_1(p)-L_1(p)\right)T_0(z)\right)\mathrm{d}\theta\mathrm{dz}=0.\end{aligned}\tag{22}$$

Because Eq. (22) always holds for an arbitrary function $T_0^*(z)$, it leads to

$$\int_0^{2\pi}\left(\Lambda_1(p)-L_1(p)\right)T_0(z)\mathrm{d}\theta=0.\tag{23}$$

After some calculation, Eq. (23) reduces to

$$\begin{aligned}\tilde{L}(p)T_0(z)&:=\frac{\sigma^2}{2}\frac{\mathrm{d}^2T_0(z)}{\mathrm{d}z^2}+\left(\Delta-\alpha\mu(z)\right)\frac{\mathrm{d}T_0(z)}{\mathrm{d}z}+p\left(Q_0+Q(z)\right)T_0(z)\\&=\Lambda_1(p)T_0(z),\end{aligned}\tag{24}$$

where

$$\mu(z)=\frac{1}{\pi}\int_0^{\pi}s_1(\theta)\cos(z+\alpha\theta)\mathrm{d}\theta,$$

$$Q(z)=\frac{1}{\pi}\int_0^{\pi}q_1(\theta)\cos(z+\alpha\theta)\mathrm{d}\theta,$$

$$Q_0=\frac{c_1}{2}+\frac{\omega(r-1)}{\pi}.$$

Therefore, the first-order perturbation solution of the moment Lyapunov exponent $\Lambda_1(p)$ is the largest eigenvalue of Eq. (24) with the associated eigenfunction $T_0(z)$. Since ε is a small parameter, and $\Lambda_0(p)=0$, the *p*th moment Lyapunov exponent $\Lambda(p)$ is approximately written as

$$\Lambda(p)\cong\varepsilon\Lambda_1(p).\tag{25}$$

Hence, the approximate analytic solution of the *p*th moment Lyapunov exponent $\Lambda(p)$ can be obtained by solving the largest eigenvalue problem of Eq. (24). And from Eq. (2), the largest Lyapunov exponent λ can be given as

$$\lambda=\frac{\mathrm{d}}{\mathrm{d}p}\Lambda(p)\bigg|_{p=0}\cong\varepsilon\frac{\mathrm{d}}{\mathrm{d}p}\Lambda_1(p)\bigg|_{p=0}.\tag{26}$$

4. Solution of the eigenvalue problem

According to Refs. [36, 46, 47], Eq. (24) can be solved by applying the method of orthogonal expansion. Based on the nature of coefficients, the eigenfunction $T_0(z)$ is given as a Fourier series, i.e.,

$$T_0(z) = u_0 + \sum_{n=1}^{\infty}\left(u_n \cos(nz) + v_n \sin(nz)\right). \tag{27}$$

Substituting Eq. (27) into Eq. (24) and integrating with respect to z yields

$$\sum_{n=0}^{\infty}\begin{pmatrix} \hat{C}_{mn} & \bar{C}_{mn} \\ \hat{D}_{mn} & \bar{D}_{mn} \end{pmatrix}\begin{pmatrix} u_n \\ v_n \end{pmatrix} = \Lambda_1(p)\begin{pmatrix} u_m \\ v_m \end{pmatrix}, \qquad m = 0,1,2,\cdots \tag{28}$$

where

$$\hat{C}_{mn} = \frac{1+\operatorname{sgn}(m)}{2\pi}\int_0^{2\pi} \tilde{L}(p)\left(\cos(nz)\right)\cos(mz)\,\mathrm{d}z,$$

$$\bar{C}_{mn} = \frac{1+\operatorname{sgn}(m)}{2\pi}\int_0^{2\pi} \tilde{L}(p)\left(\sin(nz)\right)\cos(mz)\,\mathrm{d}z,$$

$$\hat{D}_{mn} = \frac{1+\operatorname{sgn}(m)}{2\pi}\int_0^{2\pi} \tilde{L}(p)\left(\cos(nz)\right)\sin(mz)\,\mathrm{d}z,$$

$$\bar{D}_{mn} = \frac{1+\operatorname{sgn}(m)}{2\pi}\int_0^{2\pi} \tilde{L}(p)\left(\sin(nz)\right)\sin(mz)\,\mathrm{d}z.$$

The necessary and sufficient condition for the existence of a nontrivial solution to Eq. (28) is that the determinant of the corresponding coefficient matrix is zero. Thus, the problem of calculating the first-order perturbation solution of the pth moment Lyapunov exponent $\Lambda_1(p)$ is transformed into solving the largest eigenvalue of the coefficient matrix. In the present paper, a series of approximations by truncating the sums are constructed to evaluate $\Lambda_1(p)$. Consider the truncated system

$$\sum_{n=0}^{N}\begin{pmatrix} \hat{C}_{mn} & \bar{C}_{mn} \\ \hat{D}_{mn} & \bar{D}_{mn} \end{pmatrix}\begin{pmatrix} u_n \\ v_n \end{pmatrix} = \Lambda_1(p)\begin{pmatrix} u_m \\ v_m \end{pmatrix}, \qquad m = 0,1,2,\cdots N \tag{29}$$

which can be written as the following $2N+1$ dimensional system

$$\left(\mathbf{C} - \Lambda_1(p)\mathbf{I}\right)\mathbf{X} = \mathbf{0}, \tag{30}$$

where $\mathbf{I}$ is an identity matrix; $\mathbf{X} = \left[u_0, u_1, \cdots, u_N, v_1, v_2, \cdots, v_N\right]^T$;

$$\mathbf{C} = \begin{bmatrix} \left(\hat{C}_{mn}\right)_{(N+1)\times(N+1)} & \left(\bar{C}_{mn}\right)_{(N+1)\times N} \\ \left(\hat{D}_{mn}\right)_{N\times(N+1)} & \left(\bar{D}_{mn}\right)_{N\times N} \end{bmatrix}.$$

Therefore, the approximate value of the first-order perturbation solution $\Lambda_1(p)$ can be given by calculating the largest eigenvalue of the matrix $\mathbf{C}$. Through finding the leading eigenvalue of a series of submatrices, a set of approximations to $\Lambda_1(p)$ are obtained, which will converge to the true value as $N \to \infty$.

5. Parametric resonance and stochastic stability

The numerical results of pth moment Lyapunov exponent are calculated by Monte Carlo (MC) method[48], which are compared with the approximate analytical solution obtained by using the singular perturbation method. For different resonance situations, the change curves of the numerical and analytical results of pth moment Lyapunov exponent are given in Fig.1, respectively. It can be found that the approximate analytic solution converges as N increases, and the change curves of pth moment Lyapunov exponent basically coincide completely when $N \geq 2$. Furthermore, there is very little difference between the numerical and analytical results, which proves that the approximate analytical solution of pth moment Lyapunov exponent obtained by the singular perturbation method is reliable. It is sufficient as $N = 3$, and the fourth-order approximate analytical results of largest Lyapunov exponent and pth moment Lyapunov exponent are used to discuss the stochastic stability of the Rayleigh-Van der Pol vibro-impact system driven by bounded noise.

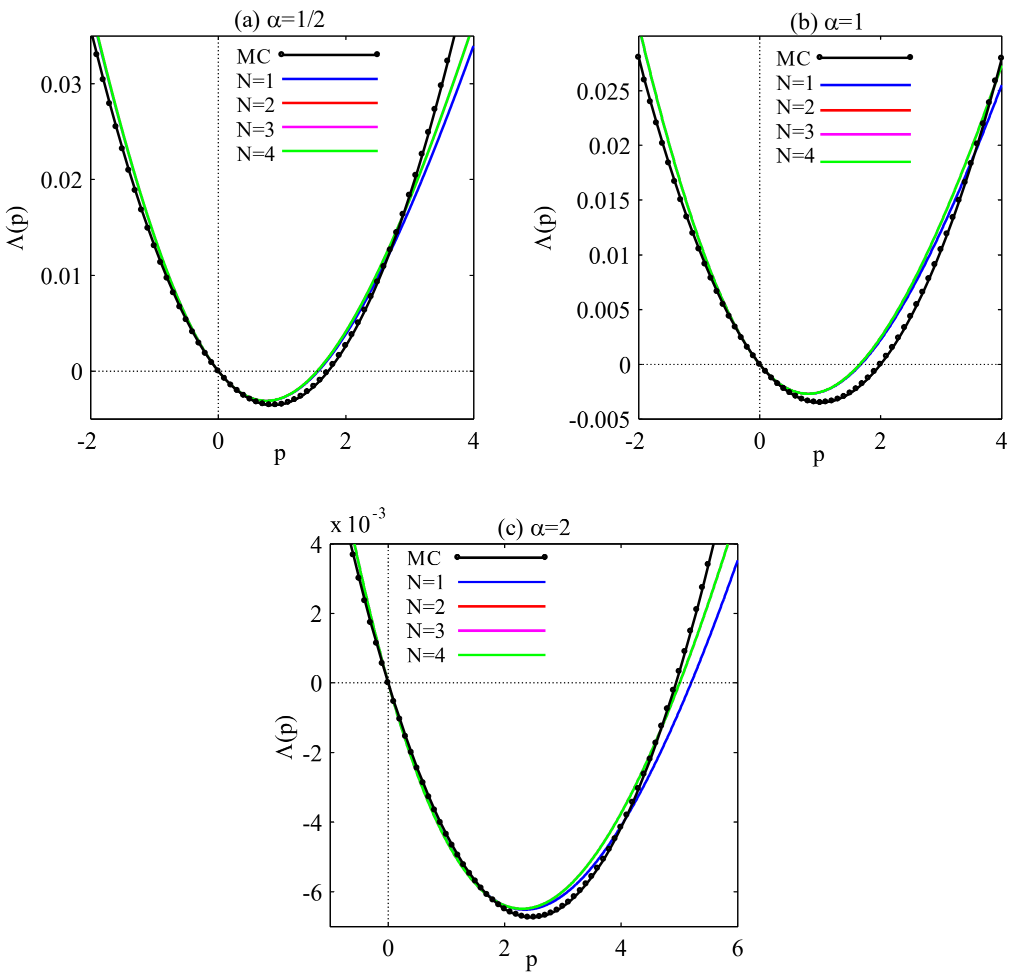


Fig.1 Numerical and analytical results of pth moment Lyapunov exponents for

$$\varepsilon = 0.1, \omega = 3, \sigma = 2, r = 0.8, c_1 = 0.2.$$

5.1 Parametric resonance

The effects of primary resonance ($\alpha = 1$), superharmonic resonance ($\alpha = 1/2$) and subharmonic resonance ($\alpha = 2$) on the stochastic stability are depicted in Fig.2 and Fig.3. Based on the fact revealed in Fig.2 and Fig.3, it is clear that the parametric resonance will significantly weaken the stochastic stability of the vibro-impact system, especially the subharmonic resonance. In this paper, we will focus on the subharmonic resonance case.

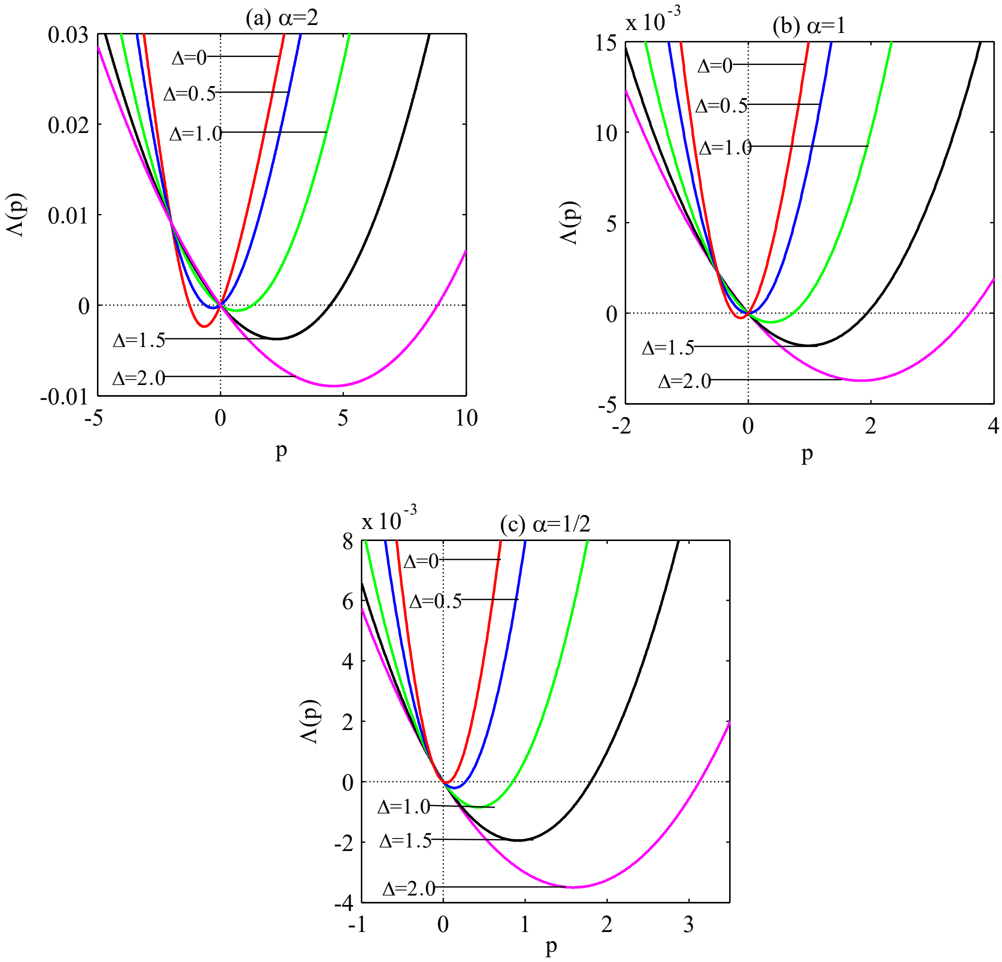


Fig.2 Effect of parametric resonance on the pth moment Lyapunov exponent for

$$\varepsilon = 0.1, \omega = 3, \sigma = 1, r = 0.9, c_1 = 0.1.$$

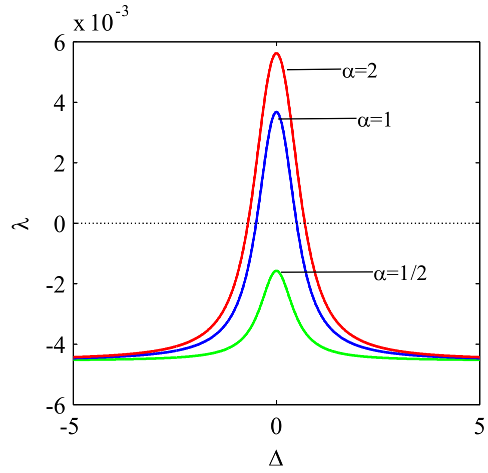
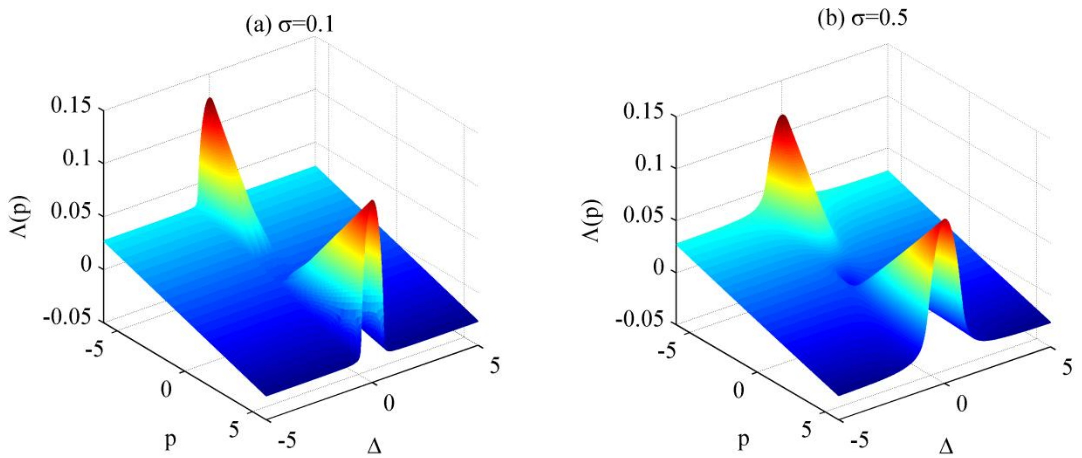


Fig.3 Effect of parametric resonance on the largest Lyapunov exponent for

$$\varepsilon = 0.1, \omega = 3, \sigma = 1, r = 0.9, c_1 = 0.1.$$

Fig.4 and Fig.5 show the influence of noise intensity σ on the parameter resonance. It can be seen in Fig.4 and Fig.5 that the noise intensity σ has a great influence on the parametric resonance of the pth moment Lyapunov exponent $\Lambda(p)$ and the largest Lyapunov exponent λ, and the resonance region gradually becomes larger along with the increase of noise intensity. The smaller the noise intensity, the larger the absolute values of the pth moment Lyapunov exponents and the largest Lyapunov exponents around $\Delta = 0$, indicating that the parametric resonance is particularly prominent. When the noise intensity is small, the influence of parametric resonance on the stochastic stability of the system is particularly significant. According to the power spectral density of a bounded noise process, the larger the noise intensity value, the wider the power spectral density, and then the wider range of noise energy distribution. Conversely, small noise intensity will lead to narrow-band process.



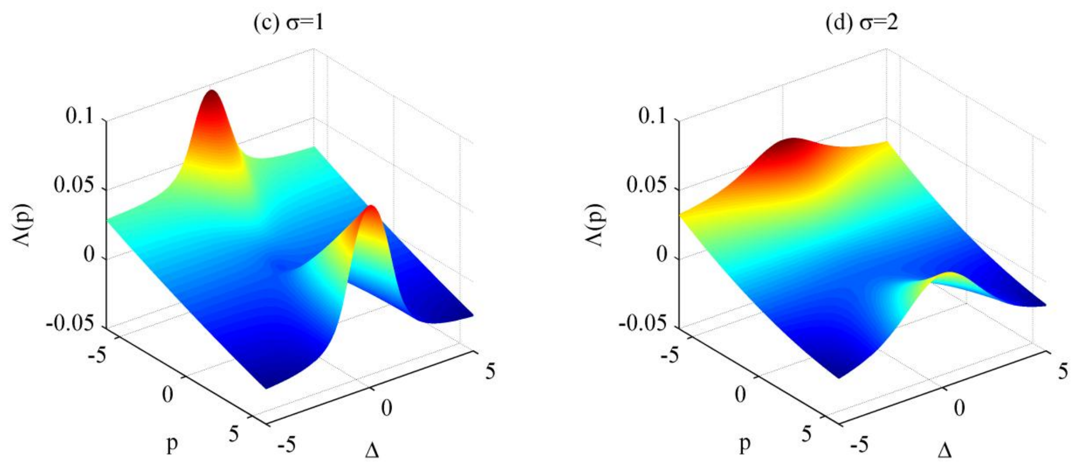


Fig.4 Effect of noise intensity and parametric resonance on the *p*th moment Lyapunov exponent for $\varepsilon = 0.1, \alpha = 2, \omega = 3, r = 0.9, c_1 = 0.1$.

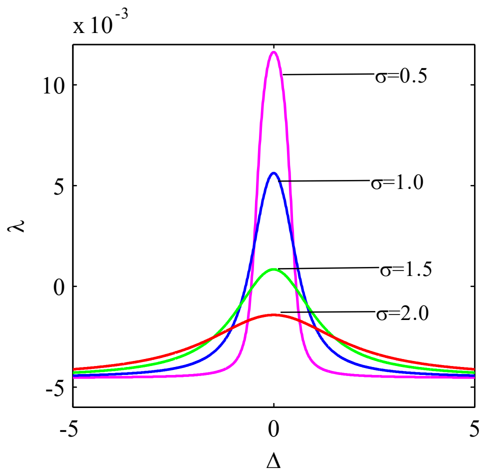


Fig.5 Effect of noise intensity and parametric resonance on the largest Lyapunov exponent for

$$\varepsilon = 0.1, \alpha = 2, \omega = 3, r = 0.9, c_1 = 0.1.$$

5.2 Stability analysis

In view of Fig.5 and Fig.6, it can be found that, the vibro-impact system becomes more and more stable as the noise intensity σ increases, and the stochastic stability gradually changes from unstable to stable. This phenomenon can be explained from the power spectral density function of the bounded noise. The larger the value of the noise intensity, the wider the frequency band of the power spectrum, which indicates that the power of the noise is more evenly distributed over the entire frequency range, and therefore the system is more stable.

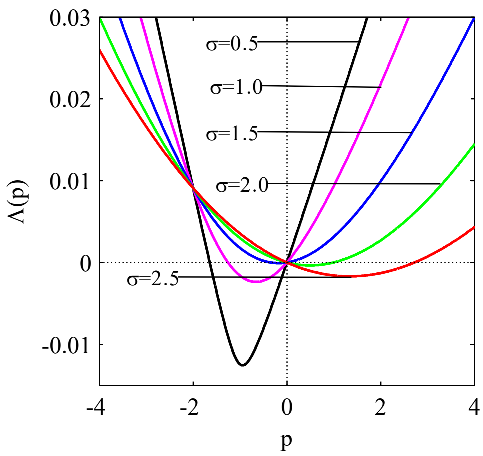


Fig.6 Effect of noise parameter σ on the pth moment Lyapunov exponent for

$$\varepsilon = 0.1, \alpha = 2, \omega = 3.0, r = 0.9, c_1 = 0.1$$

Fig.7 shows that the natural frequency ω has a significant influence on the moment stability of the vibro-impact system. With the increase of the natural frequency, the moment stability index and stability region of the system increase continuously, which indicates that the natural frequency ω can enhance the moment stability of the system. It can also be found from Fig.8 that the larger the natural frequency, the smaller the largest Lyapunov exponent λ, and the stronger the sample stability of the system. Therefore, the existence of collision factors will cause the natural frequency ω to have a significant influence on the stochastic stability of the system, and increasing the value of ω will make the random vibro-impact system more stable.

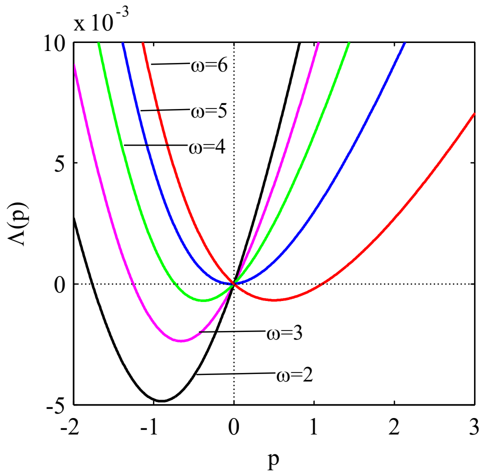


Fig.7 Effect of parameter ω on the pth moment Lyapunov exponent for

$$\varepsilon = 0.1, \alpha = 2, \sigma = 1.0, r = 0.9, c_1 = 0.1.$$

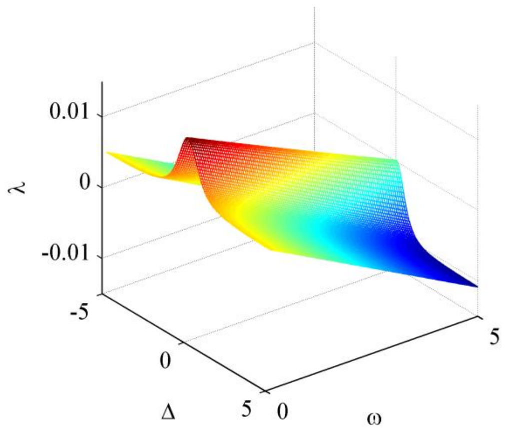


Fig.8 Effect of parameter ω on largest Lyapunov exponent for

$\varepsilon = 0.1, \alpha = 2, \sigma = 1.0, r = 0.9, c_1 = 0.1$.

The effects of recovery coefficient r on the *p*th moment Lyapunov exponents $\Lambda(p)$ and largest Lyapunov exponents λ are shown in Fig.9 and Fig.10, respectively. From Fig.9 and Fig.10, it is clear that with the increase of parameter r, the stochastic stability of the system gradually weakens. Thus, the recovery coefficient r will weaken the stochastic stability, and small recovery coefficient can make the random vibro-impact system more stable. However, for the sake of analysis in this paper, the energy loss of vibro-impact system is assumed a small amount (i.e., $0 \le 1 - r \ll 1$). Therefore, in practical engineering, the appropriate recovery coefficient r should be chosen to stabilize the system.

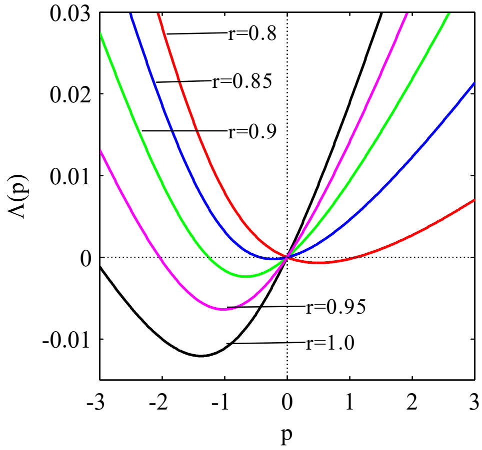


Fig.9 Effect of the recovery coefficient r on the *p*th moment Lyapunov exponent for

$\varepsilon = 0.1, \alpha = 2, \omega = 3, \sigma = 1.0, c_1 = 0.1$.

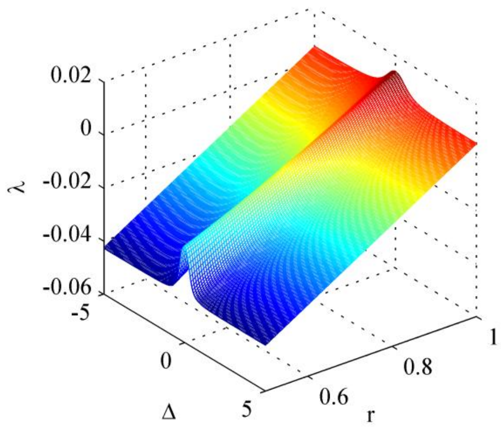


Fig.10 Effect of the recovery coefficient r on the largest Lyapunov exponent for $\varepsilon = 0.1, \alpha = 2, \omega = 3, \sigma = 1.0, c_1 = 0.1$.

Based on the fact revealed in Fig.11 and Fig.12, it can be found that the damping coefficient c_1 has a great influence on the moment stability and sample stability of the vibro-impact system, and reducing the value of c_1 will make the Rayleigh-Van der Pol vibro-impact system more stable.

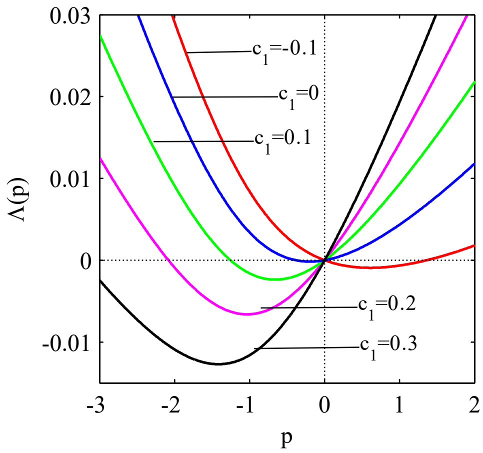


Fig.11 Effect of parameter c_1 on the pth moment Lyapunov exponent for $\varepsilon = 0.1, \alpha = 2, \omega = 3, \sigma = 1.0, r = 0.9$.

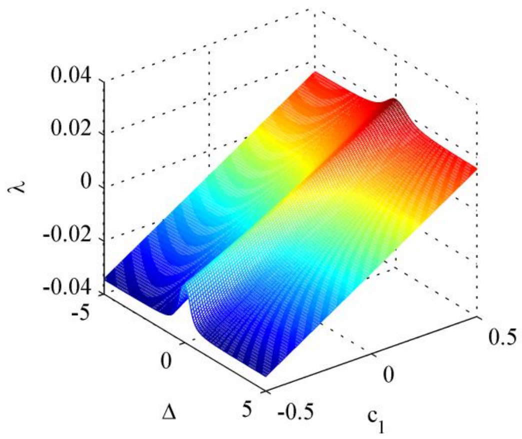


Fig.12 Effect of parameter c_1 on the largest Lyapunov exponent for
$\varepsilon = 0.1, \alpha = 2, \omega = 3, \sigma = 1.0, r = 0.9$.

6. Conclusion

In the present paper, for a Rayleigh-Van der Pol vibro-impact system excited by bounded noise, the parametric resonance (primary resonance, superharmonic resonance and subharmonic resonance) is studied, and the stochastic stability is discussed by calculating the *p*th moment Lyapunov exponent. The vibro-impact system is transformed into a smooth dynamic system through utilizing the Zhuravlev transformation. As for finite p values and weak noise excitation, the first-order approximate analytical expression of the *p*th moment Lyapunov exponent $\Lambda(p)$ is obtained by using the method of singular perturbation and Fourier series expansion. Then, the largest Lyapunov exponent λ of the vibro-impact is determined via applying the relationships with $\Lambda(p)$. Furthermore, the accuracy of the approximate analytical results obtained by the singular perturbation method is verified by comparing the Monte Carlo numerical results of the *p*th moment Lyapunov exponent with the approximate analytical results. Finally, based on the results of $\Lambda(p)$ and λ, the effects of the parametric resonance (primary resonance, superharmonic resonance and subharmonic resonance) and the different system parameters on the stochastic stability of the vibro-impact system are investigated, and some results are presented. The effect of noise intensity σ on the parameter resonance is also discussed.

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