13

$\mathcal{M}_{\mathcal{B}\mathcal{B}'}$ y $\mathcal{M}_{\mathcal{B}'\mathcal{B}}$

obtener $\text{Coord}_{\mathcal{B}'}(\text{Coord}_{\mathcal{B}}(2,-1,1))$.

a) $\mathcal{B} = \{(0,1,1), (1,0,1), (1,1,0)\}$

$\mathcal{B}' = \mathcal{C} \equiv$ canónica de $\mathbb{R}^3$

$$\mathcal{M}_{\mathcal{B}\mathcal{C}} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

coordenadas de los vectores de $\mathcal{B}$ en $\mathcal{C}$, como es la canónica son ellos mismos

$$\mathcal{M}_{\mathcal{C}\mathcal{B}} = \mathcal{M}_{\mathcal{B}\mathcal{C}}^{-1}.$$

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \Rightarrow |A| = (1+1) - (0) = 2$$

$$\text{Adj}(A) = (-1)^{i+j} \det(A_{ij})$$

menor 2×2 fruto de eliminar fila i y columna j

$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1$

$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = 1$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 1$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$

$$\text{Adj}(A) = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\text{Adj}^t(A) = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\frac{1}{|A|} \cdot \text{Adj}^t(A) = \begin{pmatrix} -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{pmatrix}$$

Comprobación :

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$ Idem con el otro sentido

Así $\mathcal{M}_{C\mathcal{B}} = \begin{pmatrix} -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{pmatrix}$

$$\mathrm{Coord}_{\mathcal{B}}(2,-1,1) = \begin{pmatrix} -1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} =$$

$$= \left(\frac{-2}{2} - \cancel{\tfrac{1}{2}} + \cancel{\tfrac{1}{2}},\ \frac{2}{2} + 1/2 + 1/2,\ \frac{2}{2} - 1/2 - 1/2 \right) =$$

$$= \underline{(-1, 2, 0)}$$

b) $\mathcal{B} = C$

$\mathcal{B}' = \{(0,0,1), (0,1,1), (1,1,1)\}$

$$\mathcal{M}_{\mathcal{B}'C} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$\mathcal{M}_{C\mathcal{B}'} = \mathcal{M}_{\mathcal{B}'C}^{-1}$ ← Como antes.

$$\mathrm{Coord}_{\mathcal{B}'}(2,-1,1) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = (-1, -2, 0)$$