Ej 8:

$A = \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix}$ $\quad p(\lambda) = \det(A - \lambda Id) = \begin{vmatrix} -1-\lambda & 1 \\ 2 & -\lambda \end{vmatrix} =$

$= (-1-\lambda)(-\lambda) - 2 = (1+\lambda)\cdot\lambda - 2 =$

$= \lambda^2 + \lambda - 2 = 0 \quad \lambda = \frac{-1 \pm \sqrt{1+4\cdot 2}}{2} =$

$= \frac{-1 \pm \sqrt{9}}{2} =$

$= \frac{-1 \pm 3}{2} <$ $\frac{-1+3}{2} = \frac{2}{2} = 1$, $\frac{-1-3}{2} = \frac{-4}{2} = -2$

$p(\lambda) = (\lambda - 1)(\lambda + 2)$

$\sigma(A) = \{1, -2\}$ como son ambas raíces reales simples $\Rightarrow$

A diag. $D = \begin{pmatrix} +1 & 0 \\ 0 & -2 \end{pmatrix}$. Calculemos la matriz de paso

$V_1 = \{ (x,y) \in \mathbb{R}^2 \mid (A - 1 Id_2)\cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \} =$

$\overset{\shortparallel}{\vec{v}}$

$= \{ (x,y) \in \mathbb{R}^2 \mid \begin{pmatrix} -2 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \}$

$(x,y) \in V_1 \Leftrightarrow \left.\begin{matrix} -2x + y = 0 \\ 2x - y = 0 \end{matrix}\right\} \Rightarrow 2x = y \Rightarrow x = y/2$

Así $(x,y) \in V_1 \Leftrightarrow (y/2, y) = y(\frac{1}{2}, 1) \Rightarrow$

$\boxed{V_1 = \langle (1/2, 1) \rangle.}$

$V_{-2} = \{ (x,y) \in \mathbb{R}^2 \mid (A + 2 Id_2)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \} =$

$= \{ (x,y) \in \mathbb{R}^2 \mid \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \} \Rightarrow$

$(x,y) \in V_{-2} \Rightarrow \begin{matrix} x + y = 0 \\ 2x + 2y = 0 \end{matrix} \Rightarrow x = -y$ así

$(x,y) \in V_{-2} \Rightarrow (-y, y) = y(-1, 1) \Rightarrow \boxed{V_{-2} = \langle (-1, 1) \rangle}$