Design of a nonlinear SITR fractal model based on the dynamics of a novel coronavirus (COVID-19)

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Abstract:

The aim of the present paper is to state a simplified nonlinear mathematical model to describe the dynamics of the novel coronavirus (COVID-19). The design of the mathematical model is described in terms of four categories susceptible (S), infected (I), treatment (T) and recovered (R), i.e., SITR model with fractals parameters. These days there are big controversy on if is needed to apply confinement measure to the population of the word or if the infection must develop a natural stabilization sharing with it our normal life (like USA or Brazil administrations claim). The aim of our study is to present different scenarios where we draw the evolution of the model in 4 different cases depending on the contact rate between people. We show that if no confinement rules are applied the stabilization of the infection arrives around 300 days affecting a huge number of population. On the contrary with a contact rate small, due to confinement and social distancing rules, the stabilization of the infection is reach earlier.

Keywords: SITR model; Coronavirus; Adams numerical results; Diseases; Treatment.

1. Introduction

The human life is affected of many diseases of several level of lethality from its origin. To mention a few of them, Ebola is an infectious deathly disease that transmits in humans from the diseased animals, such as non-human primate or a fruit bat that has killed many people all around the world. HIV is another deathly world disease that has its genesis from crossed species from chimps to humans. This transferred disease was unknown in the eight decades of the 19th century and visible signs or symptoms did not accompany a transmission. HIV reported in five countries and this disease infected almost 300,000 persons. Lassa fever is considered very dangerous and is believed to convert in humans from rates. Many other serious diseases create between the interaction of other living-beings and humans. Therefore, humans have established several medical operates and used many measures to prevent and to cure, where possible, some of the above reported diseases.

The coronavirus (COVID-19) is a spreading virus causing respiratory infection and highly transferred from human to human through small droplets from the mouth or nose, which spread from an infected person exhales or coughs [1]. These drops land on surfaces and objects near the persons and one can catch this virus by touching their mouth, nose or eyes. The novel COVID-19 was reported in the Wuhan city located at the Hubei province of China on December 31, 2019 [2].

This virus killed many persons and currently it become a big challenge for the whole world to control it. The number of COVID-19 cases are increasing day by day, which are not reported only in the poor countries, but also from the developed countries. Currently, almost every country and continent of the world is not avoided from this dangerous virus. Many people killed in China from this serious disease, but China did lockdown and they controlled this virus to some extent. The numbers of cases in the developing countries are reported in thousands and the deaths in Italy and Spain has been increased from China.

The major symptoms of COVID-19 are tiredness, dry cough and fever. A few patients may have pains and aches, runny nose, nasal congestion, diarrhea and sore throat. These mentioned symptoms are normally mild and begin slowly. Most of the people (around 80%) infected with the COVID-19 have been recovered without special treatment. One infected person out of six from COVID-19 gets seriously ill and feels difficulty in breathing. This virus affects badly on the older people, and those who facing medical serious illness like diabetes, cardiovascular disease, cancer and chronic respiratory disease. The people having fever, difficulty in breathing and cough should pursue medical attention.

The COVID-19 is spreading day by day and many questions arises in the mind, how much time will it take to make the medicine or vaccine that can control this disease? When the world can get rid of this deathly virus? How much financial crisis and human loss will the world face. If this virus gets an end then can it spread again? Not yet. To date, there is no answer of any of the above question. The number of corona virus cases has been reported 438441, the recovered persons are 111877 and the number of deaths reported are 19656. However, the recovery ratio is larger in number as compared to death ratio, but the number of deaths is increasing day by day. The number of deaths occur from COVID-19 in the last ten days (March, 15-24) graphically presented

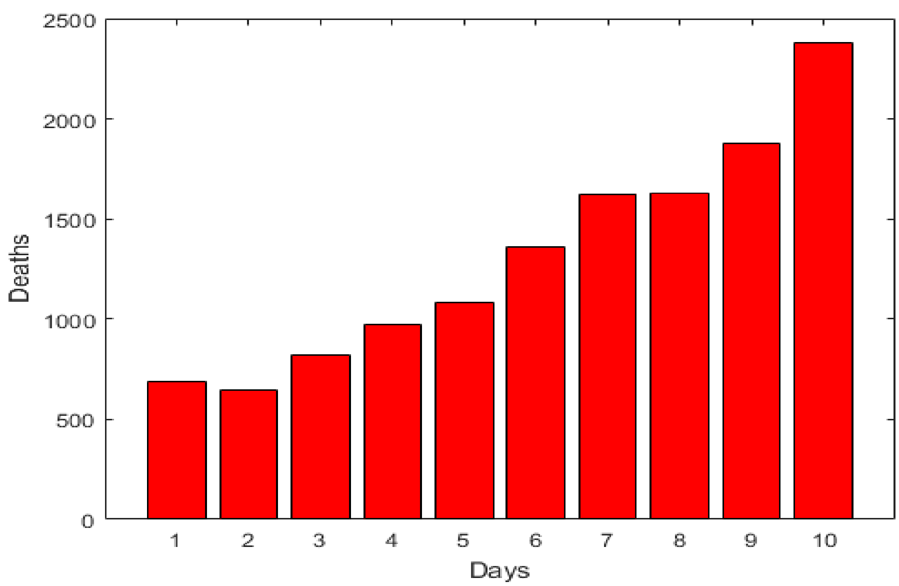

Fig.1: The number of deaths reported through COVID-19 for the last ten days (March, 15-24)

The aim of the present study is to design a model of SITR type based on the novel COVID-19 using the current information that we have. A numerical solution of the SITR model have been presented using the Adams method [3-13] which work good for model where the values of the parameters are not 100% accurate like in this case.

The novel COVID-19 interaction that we modelized in a simplify way is based on four mechanisms: susceptible (*S*), infected (*I*), treatment (*T*) and recovered (*R*). The susceptible (*S*) is further divided in two categories $S_1(\tau)$ and $S_2(\tau)$. The first category $S_1(\tau)$ indicates those individuals, who are yet not infected from the COVID-19, while the second category $S_2(\tau)$ represents those individual who are also not infected yet but these individuals are older or have some serious diseases. $I(\tau)$ represents the infected people from the COVID-19, which are infected with this serious disease at the time τ. However, proper treatment in the form of medicine, drugs or vaccination of this virus yet not discovered, but one can get rid of using some precautionary measures, so $T(\tau)$ is used for the treatment of this virus. The fourth factor is used for the recovery $R(\tau)$ of those individuals who recovered from this serious disease at time τ by using these precautionary measures and it is noticed that a large number of such category exist. The general form of the designed SITR model is given as:

$$\begin{cases} \dfrac{dS_1(\tau)}{d\tau} = B - \beta I(\tau)S_1(\tau) - \delta\beta T(\tau) - \alpha S_1(\tau), & S_1(0) = I_1, \\ \dfrac{dS_2(\tau)}{d\tau} = B - \beta I(\tau)S_2(\tau) - \delta\beta T(\tau) - \alpha S_2(\tau), & S_2(0) = I_2, \\ \dfrac{dI(\tau)}{d\tau} = -\mu I(\tau) + \beta I(\tau)\left(S_1(\tau) + S_2(\tau)\right) + \beta\delta T(\tau) - \alpha I(\tau) + \sigma I(\tau), & I(0) = I_3, \\ \dfrac{dT(\tau)}{d\tau} = \mu I(\tau) - \alpha T(\tau) - \rho T(\tau) + \varepsilon T(\tau) + \psi T(\tau), & T(0) = I_4, \\ \dfrac{dR(\tau)}{d\tau} = -\alpha R(\tau) + \rho T(\tau), & R(0) = I_5. \end{cases} \quad \textbf{(1)}$$

The above model shows represent SITR model with fractals parameters. The state variables along with related descriptions are given in the Table below as:

Table 1: State variables definitions of the SITR model.

Parameter	**Description**
$S_1(\tau)$	Not Infected individuals
$S_2(\tau)$	Not infected old age or serious diseased people
$I(\tau)$	Infected rate from the COVID-19
$R(\tau)$	Recovered rate from the COVID-19
$T(\tau)$	Treatment
B	Natural birth rate
β	Contact rate
δ	Reduce infection by treatment
α	Death rate
μ	Recovery rate

σ	Fever, dry cough and tiredness rate
ρ	Infection rate for treatment
ε	Sleep rate
ψ	Healthy food rate
$I_i, i = 1, 2, 3, 4, 5$	Initial conditions

The construction of the mathematical model simplifies parameters, assumptions and variables. To examine the mathematical form of the disease of the infection, many numerically and analytical investigations have been implemented. Many researchers [14-17] worked on the epidemic models, as well as valuations on theoretical progresses. Ogren and Martin [18] applied the embedded Newton's scheme to obtain the optimal control strategy in the biological SIR system. Goufo et al [19] implemented fractional model based on SIR epidemic for temporal and spatial spread of measles in populations. Mickens [20] worked on the vaccination in the system of discrete-time for periodic viruses spread. Moreover, some other epidemic system has also been studied and their references are therein [21-23]. The present study has the aim of solving the model (1) numerically by using the Adams method by adjusting the best parameter values.

The reaming parts of this study are organized as follows: In section 2, the detailed discussions based on the present results for solving the SITR system based on the COVID-19 model are presented. The 3rd section indicates the conclusions and future research direction of the present study.

2. Results and discussions

In this section, the detailed discussions based on the present results for solving the designed SITR model based on the COVID-19 are presented. The state variables of the SITR model are provided and the analysis of contact rate is provided in the figures.

2.1 SITR Model

Two different parameter values have been indicated in the Table 2 that shows the results of each category based on the SITR model. The model (1) using different epidemic parameter values is written as:

Table 2: State variables definitions of the SITR model.

Parameter	Values
B	0.3
β	0.35,0.25,0.3 and 0.1
δ	0.3
α	0.25
μ	0.1
σ	0.005
ρ	0.3
ε	0.1
ψ	0.2
I_1	0.65
I_2	0.15
I_3	0.75

I_4	0.35
I_5	0.1

To represent the graph of all mechanisms susceptible (*S*), infected (*I*), treatment (*T*) and recovered (*R*) along with the two subcategories of susceptible is plotted in Figures 2 and 3 by taking different values of β. Figure 2 shows the plots of SITR model using the suitable epidemic parameter values given in Table 2. The variation of τ is taken 0 to 300, in appropriate units. It is seen that at $\tau = 30$, the infected (*I*), treatment (*T*) and recovery (*R*) slightly decreased and $\tau = 50$ both the susceptible S_1 and S_2 categories decreased gradually, while the other three categories infected (*I*), treatment (*T*) and recovery (*R*) increased at that rate. To move further, the increasing and decreasing behavior of all the categories is noticed up to $\tau = 90$ and then it became stable.

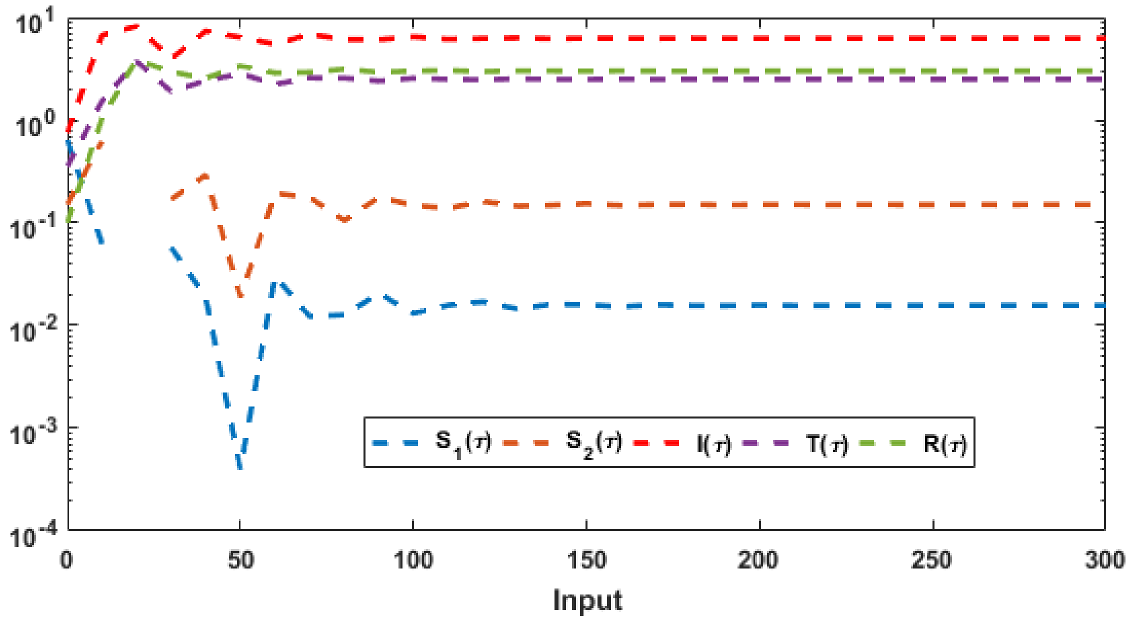

Fig. 2: The susceptible, infective and recovered rate with respect to time τ for $\beta = 0.35$

Figure 3 shows the plots of SITR model using the same values given in Table 2 for $\beta = 0.3$. It is observed that at $\tau = 30$ and 52, the susceptible S_1 and S_2 categories decreased, while at that rate the other three categories infected (*I*), treatment (*T*) and recovery (*R*) increased. Moreover, at $\tau = 47$, the susceptible categories increased, while other three categories decreased and it sooner becomes stable.

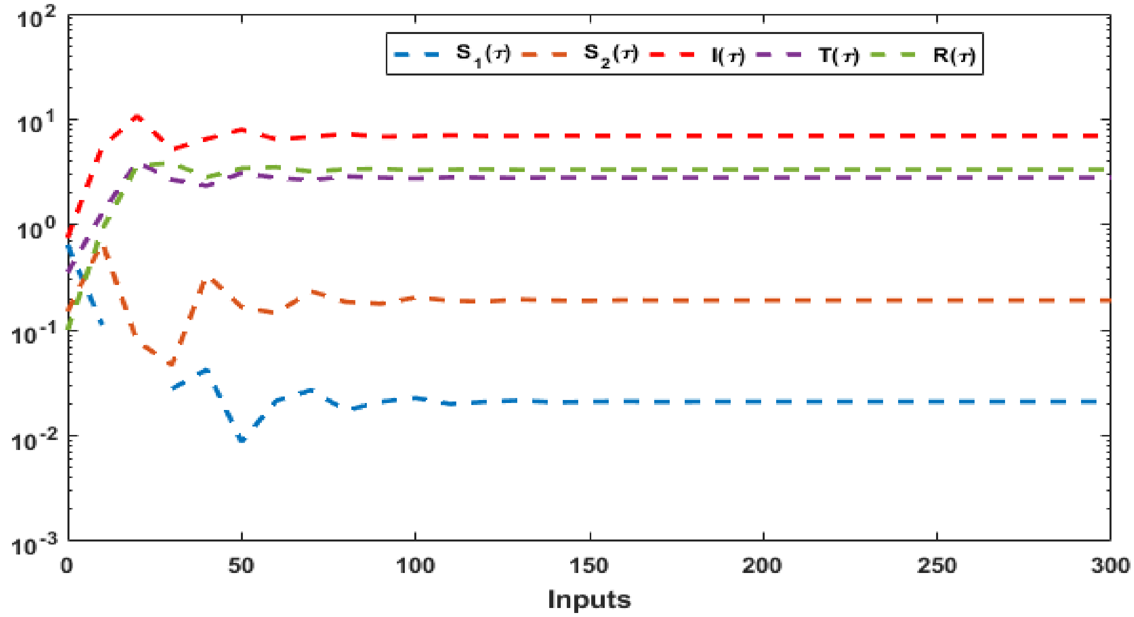

Fig. 3: The susceptible, infective and recovered rate with respect to time τ for $\beta = 0.3$

Figure 4 shows the plots of SITR model using the same values given in Table 2 for $\beta = 0.25$. It is seen that at $\tau = 30$, the susceptible S_1 and S_2 categories decreased gradually, while the other three categories infected (*I*), treatment (*T*) and recovery (*R*) increased at that rate. Moreover, at $\tau = 45$, the susceptible categories increased, while other three categories decreased and it sooner become stable.

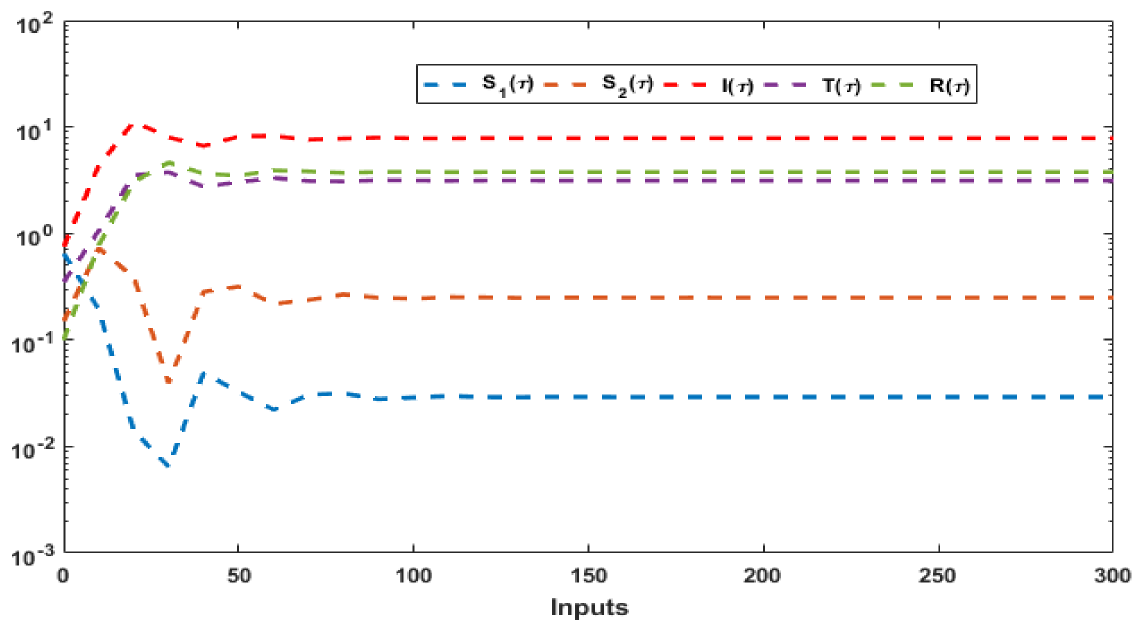

Fig. 4: The susceptible, infective and recovered rate with respect to time τ for $\beta = 0.25$

Figure 5 indicates the plots of SITR model using the same values given in Table 2 for $\beta = 0.1$. It is observed that all the categories become stable in 20 days.

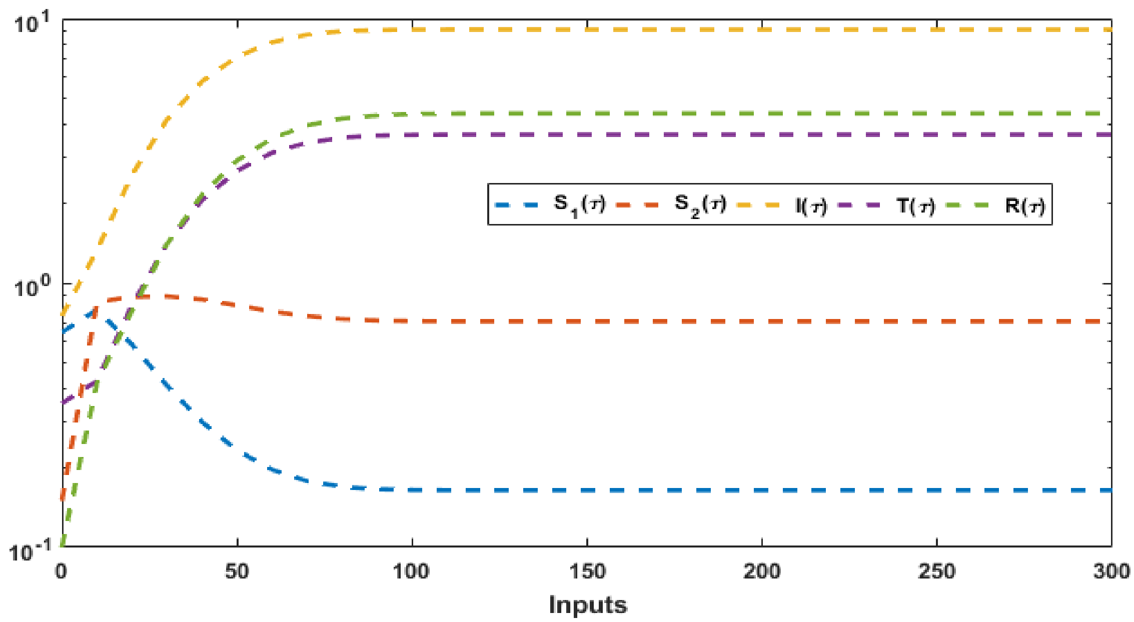

Fig. 5: The susceptible, infective and recovered rate with respect to time τ for $\beta = 0.1$

4. Conclusion

We provided a SITR model which described in a simplified way the dynamics of COVID-19. The designed model is classified into 4 categories named as susceptible (*S*), infected (*I*), treatment (*T*) and recovered (*R*) at time τ. Moreover, the category susceptible (*S*) is further divided in two subcategories $S_1(\tau)$ and $S_2(\tau)$. We proof that the confinament rules are essential to control the situation in a reasonable time. If we reduce the contact rate between people we obtain stabilization of classes much sooner than for bigger values of contract rate. So, we can claim that there is no chance for goverments to not apply the social distance between people.

6. Statements:

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Conflict of Interest: The authors declare that they have no conflict of interest. All authors have worked in an equal sense to obtain these results.

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