A soft computing scaled conjugate gradient procedure for the fractional order Majnun and Layla romantic story

Zulqurnain Sabir[1,2], Juan L G Guirao[3,4]
[1]Department of Mathematics and Statistics, Hazara University, Mansehra, Pakistan
[2]Department of Computer Science and Mathematics, Lebanese American University, Beirut, Lebanon
[4]Department of Applied Mathematics and Statistics, Technical University of Cartagena, Hospital de Marina, Cartagena 30203, Spain
[5]Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

Abstract: The current study shows the numerical performances of the fractional order mathematical model based on the Majnun and Layla (FO-MML) romantic story. The stochastic computing numerical scheme based on the scaled conjugate gradient neural networks (SCGNNs) is presented to solve the FO-MML. The purpose to provide the solutions of the fractional derivatives is to achieve more accurate and realistic performances of the FO-MML romantic story model. The mathematical model is divided into four dynamics, while the exactness is authenticated through the comparison of obtained and reference Adam results. Moreover, the negligible absolute error enhances the accurateness of the stochastic scheme. Fourteen numbers of neurons have been taken and the information statics is divided into authorization, training, and testing, which are divided in 12%, 77% and 11%, respectively. The reliability, capability, and accuracy of the stochastic SCGNNs is performed through the stochastic procedures using the regression, error histograms, correlation, and state transitions for solving the mathematical model.

Keywords: Fractional order; Majnun and Layla; Mathematical system; Scaled conjugate gradient; Neural networks; Reference results.

1. Introduction

Psychology is the scientific learning processes of the mind according to some American reports [1-4]. Few scientists and psychologists have been presented the mathematical relationship in the form of a love story [5-7]. The psychological applications have been noticed in social nature, human development, health, clinical actions and cognitive. One of the psychological questions comes in the mind, what are the meanings of love? Everyone has its own meanings. In general, love is divided into two categories, spiritual and physical love. The spiritual love is realistic and does not alter according to the situation, while the physical love shows an organic attraction. Love is often maddening and sometimes become terrifying when someone is involved in love. Those who in true love feel that the earth is spinning. A person falls in love without any judgement of color, religion, and creed. These emotions are not specific for the humans, but other creatures also have the feelings of love. There are various historical real love stories, some of them are Heer Ranjha, Sassi Punnuh, Sohni Mahiwal, Romeo Julliet and Layla Majnun.

The present work shows the numerical solutions of the historical fractional order mathematical model based on the Majnun and Layla (FO-MML) romantic story. This story was recognized in Persia in the 9th century. The theme of this story has also been presented in different languages including Arabic, Persian, Pakistani, Indian, and Turkish. This love story is not only reported in the literature, but these characters have been presented in various dramas and films. One of the super hit Hindi films “Aaja Nachle” is presented based on the love story of Layla-Majnun, which is directed by Anil Mehta and produced by Aditya Chopra. In this film, when the Majnun has been beaten by his master then Layla faced same wounds. Some believe that this is magic because Layla's blood was shed when the Majnun's hand harmed her. According to people, it was the gift from God or something to do with a ghost or unholy magic. Her family forbids her from being close to him. They employed various techniques to expel the evil, such as attaching a black cord to his arm. The childhood converted into the youth with their growing love, and they promised to set such a true love that will be memorized in the minds of people for centuries. The Layla’s brother Tabraiz was unhappy, he forcibly stopped Layla to meet him. He tried to kill him, but Majnun was powerful, he snapped the weapon and killed Tabraiz. The court decided to kill Majnun with stones until his last breath. Layla tried to stop, but all in vain and then Majnun got unconscious. She requested do not hit Majnun, I will be married where my family wants, but with that condition to forgive Majnun and put him out of the city. I left my heartbeat for you (Majnun). I swear no one can touch me, if someone did, he will find the clay pot instead of body. The people will weep and repent when God will come to our graves with flowers. She wedded and scarified herself. When her husband tried to touch, she resisted with these words that I am only for Majnun. Her husband ordered to his army chief to prepare against Majnun, I want to see her lover and her friendship. She caught the sward of her husband, injured herself and cried “see the blood of my hand, you cannot imagine how much we love each other”. Her husband started fight with Majnun and shouted “Majnun you are high or my castle”. Majnun replied, you are the husband of Layla, why are you shouting? Why are you expected to be a lover if you do not understand what love is? Layla is my love, life, and heartbeat. You married Layla, but you don't understand the significance of love. Majnun was asked a few questions by her husband, including: Is she your cremation? Majnun replied: she is my faith. Do you respect her? Majnun retorted, she is everything. I will cut your head; I will kill you today. Majun replied that Layla will pass away with me. I am going to keep her inside and put her in the deep well. Majnun claimed that no matter how much you dig the well, you cannot reach her. It would be better if you killed me because if I died, she would also die. He had two choices: let it go from your heart or accept her passing. Majnun accepted the second option. Her husband used a sword to kill him, but he is unaware that Layla will also receive the same wound. Both scarified to God on the same time. Nobody can protect themselves from love, even it can be Zuleka, Ghalib, Iqbal, Bulay Shah and Khusro.

We learned the value of love from all the major religions. Whether it is the Quran, the Geeta, the Bible, or the Torat, has taught us the importance of love. The holy prophet Muhammad (S.A.W) said: “Loving humanity is second only to having faith in Allah as one of the best deeds in Islam”.

Several models based on the complex variables have been used in various applications, e.g., optical systems [8], plasma physics [9], high-energy accelerators [10], rotor dynamics [11], and some other areas [12–15]. The motive of this research is to present the numerical solutions of FO-MML by applying the stochastic computing performances of the scaled conjugate gradient neural networks (SCGNNs). The time-fractional based derivatives are used in various real-world applications [16-17]. The stochastic procedures based on the artificial neural networks have been applied to solve various stiff, complicated, singular, nonlinear, linear, and grim differential systems [18-20]. Some noteworthy applications of the stochastic solvers are COVID-19 model [21-23], food chain model [24], smoking models [25], HIV systems [26], delayed differential model [27], and forth order differential model [28]. However, the stochastic SCGNNs paradigms have never been implemented to solve the FO-MML. Few novel characteristics of the stochastic method to solve the FO-MML are shown as:

- The integer model based on the Majnun and Layla model is converted into the FO-MML to find more realistic results.
- The stochastic solvers have not been applied before to solve the FO-MML love story model.

- The artificial stochastic procedures together with the SCGNNs are accessible to stimulate the FO-MML love story system.
- The study of mathematical system is presented for three variations based on the fractional kind of the mathematical Majun Layla model.
- The correctness of the scheme is observed by using the comparison of the obtained and reference results.
- The absolute error (AE) authenticates the accuracy of solutions based on the fractional kind of the mathematical Majun Layla model.
- The regression measures, error histogram (EHs) performances, correlation presentations and state transitions (STs) standards approve the dependability of proposed SCGNNs for FO-MML love story model.

The other sections of the paper are presented as: Section 2 shows the fractional order mathematical Majnun Layla model. Section 3 presents the SCGNNs performances. Section 4 signifies the simulations of FO-MML love story model, and the conclusions are given in last Section.

2. Fractional order Majnun Layla model

This section provides the FO-MML romantic story along with the real and complex relationships. The simple form of the model along with its two types of the complex variables is provided as [29-30]:

$$\begin{cases} \dfrac{dM(u)}{du} = \lambda_a + \lambda_c M(u) + L^2(u), & M_0 = c_1, \\ \dfrac{dL(u)}{du} = \lambda_b + \lambda_d L(u) + M^2(u), & L_0 = c_2, \end{cases} \quad \textbf{(1)}$$

where $\lambda_c < 0$, $\lambda_a > 0$, $\lambda_b < 0$ and $\lambda_d < 0$, while c_1, c_2, c_3 and c_4 are the initial conditions. The feelings of the Majnun and Layla have been expressed in the form of $M(u)$ and $L(u)$. λ_a and λ_b are the parameter constants that have been used to indicate the environmental properties based on their spirits. λ_a is taken as positive and fixed, which represents the sympathy and kindness for the Majnun. $\lambda_b < 0$ means the unkind behavior of the people with Layla. M^2 and L^2 indicate their love at extreme level. $\lambda_b < 0$ and $\lambda_d < 0$ indicate the true love along with their feelings. The updated form of Eq. (1) is achieved by taking the complex values $M(u) = M_r(u) + iM_i(u)$ and $L(u) = L_r(u) + iL_i(u)$ as [30]:

$$\begin{cases} \frac{d}{du}M_r(u) = \lambda_a + \lambda_c M_r(u) - L_i^2(u) + L_r^2(u), & (M_r)_0 = c_1, \\ \frac{d}{du}M_i(u) = \lambda_c M_i(u) - 2L_i(u)L_r(u), & (M_i)_0 = c_2, \\ \frac{d}{du}L_r(u) = \lambda_b + \lambda_d L_r(u) - M_i^2(u) + M_r^2(u), & (L_r)_0 = c_3, \\ \frac{d}{du}L_i(u) = \lambda_d L_i(u) + 2M_i(u)M_r(u), & (L_i)_0 = c_4, \end{cases} \tag{2}$$

where, $M_i(u)$ and $L_i(u)$ are the emotions of Majnun and Layla using the imaginary parts, while $M_r(u)$ and $L_r(u)$ are their real feelings. This work presents the fractional order mathematical Majnun and Layla (FO-MML) romantic story by applying the procedures of SCGNNs. The FO-MML based on the love relationship is written as:

$$\begin{cases} \frac{d^\upsilon}{du^\upsilon}M_r(u) = \lambda_a + \lambda_c M_r(u) - L_i^2(u) + L_r^2(u), & (M_r)_0 = c_1, \\ \frac{d^\upsilon}{du^\upsilon}M_i(u) = \lambda_c M_i(u) - 2L_i(u)L_r(u), & (M_i)_0 = c_2, \\ \frac{d^\upsilon}{du^\upsilon}L_r(u) = \lambda_b + \lambda_d L_r(u) - M_i^2(u) + M_r^2(u), & (L_r)_0 = c_3, \\ \frac{d^\upsilon}{du^\upsilon}L_i(u) = \lambda_d L_i(u) + 2M_i(u)M_r(u), & (L_i)_0 = c_4, \end{cases} \tag{3}$$

where υ signifies the FO Caputo derivative. The values of FO have been chosen between 0 and 1. The idea to apply the fractional derivatives is to obtain the precise and accurate solutions. In the fractional kind of systems, the minute particulars (super-fast and low evolution) are examined, which shows more details of the system's dynamics based on the fractional calculus that is difficult to handle by applying the integer order. The derivatives based on the fractional order performed more competency as compared to integer order with the obtainability of the condition. The fractional derivatives are used to validate the system's performance using the real applications [31-32]. Furthermore, the fractional derivatives are widely examined to solve various applications that arises in engineering, control networks, and mathematical models. The fractional calculus is widely applied over the past 3 decades by using the substantial operators, like Caputo [33], Weyl-

Riesz [34], Riemann-Liouville [35], Grnwald-Letnikov [36] and Erdlyi-Kober [37]. All above mentioned operators have their own implications. Whereas the Caputo derivative are used to solve the non-homogeneous/homogeneous initial conditions and considered easy in implementations. By keeping the importance of these fractional calculus applications, authors are motivated to perform the numerical solutions of the FO-MML by using the designed stochastic SCGNNs.

3. Proposed scheme: SCGNNs

The current section presents the designed SCGNNs to solve the FO-MML story model. The necessary procedures are presented to describe the stochastic SCGNNs along with its implementations. Fig. 1 shows the optimization measures using the multi-layer structure of SCGNNs, which are divided into two parts, mathematical model, and simulation performances. The designed operator performances are provided in two steps

i) The significant measures using the proposed SCGNNs.

ii) The execution through the stochastic SCGNNs framework is presented to solve the FO-MML story model.

The numerical measures are executed with the default parameter setting to produce the dataset. Thirteen neurons have been selected with the data selection to solve the FO-MML story model. The artificial intelligence aptitudes using the supervised learning based on the SCGNNs have been provided with best collaboration in the directories, including intricacy, overfitting, premature convergence, and underfitting performances. In addition, the system's parameters are adjusted after comprehensive simulations, knowledge, experience, and small disparities in the networks.

The stochastic SCGNNs performances are implemented in the 'Matlab' software by using 'nftool' command to get the selection of appropriate hidden neurons, learning methods, testing measures, and verification. While the execution of SCGNNs for solving the FO-MML model together with the parameter setting is given in Table 1. The training of the networks is performed through the SCGNNs, and the backpropagation is provided to progress the Jacobian '*JoB*', i.e., mean square error (MSE) to regulate the weights together with bias variables (*B*). The amendment of the decision variables using the SCG is written as:

$JoJ = JoB \times JoB,$

$Je = JoB \times e,$

$dB = \frac{-(JJ + I \times mu)}{Je}.$

In the above system, *I* present the identity vector and *e* is the error. The parameter performances of the SCGNNs are provided in Table 1 together with minor variation/disparity/adjustment. Hence, these parameter settings will be incorporated with widespread consideration.

Table 1: Parameter values to perform the SCGNNs

Index	**Settings**
Hidden neurons	13
Decreeing Mu performances	0.1
Adaptive parameter, i.e., mu	5×10^{-03}
Fitness values (MSE)	0
Increasing Mu values	10
Maximum Mu performances	10^{11}
Maximum Epochs	450
Validation tests	12%
Layers (hidden, output, input)	Single
Minimum values of the gradient	10^{-07}
Testing measures	11%
Sample selection	Random
Training performance	77%
Generated dataset	Adam method
Stoppage standards	Default

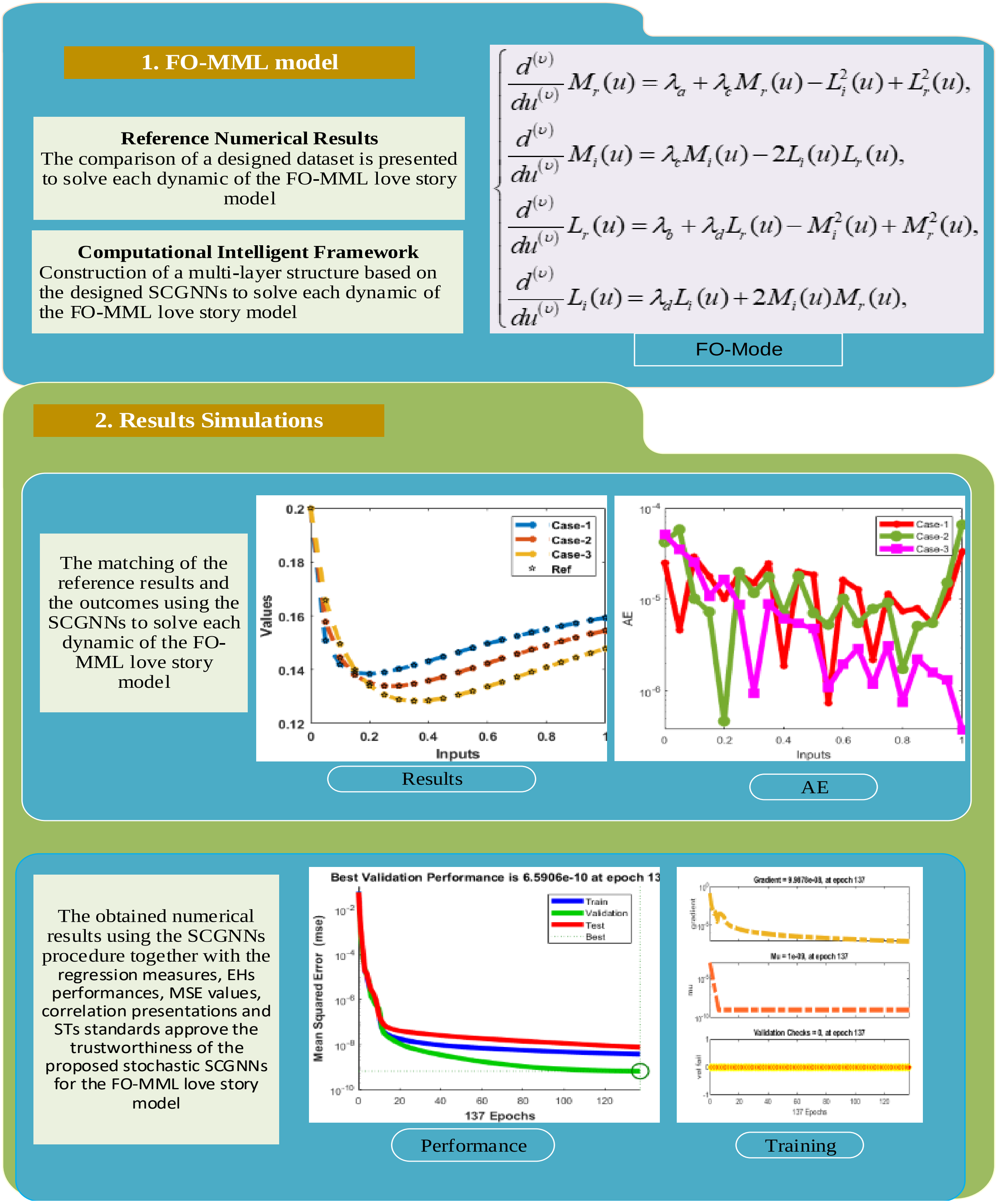


Figure 1: Workflow illustrations of scheme for FO-MML system

The numerical simulations through the stochastic SCGNNs are provided to solve the FO-MML love story model. Thirteen numbers of neurons have been provided in Fig. 2 using the statics of authorization, training, and testing, which are taken as 12%, 77% and 11% to solve the dynamics of the FO-MML. It is not always necessary to select the same statics for authorization, training, and testing. The training values have been chosen > 77% to get the improved and better presentations due to bias input performances. If the statics of training performance is < 77%, then the precision of stochastic SCGNNs is degraded suggestively. Therefore, the sample statics for the interval based on the unbiased and bias values should be selected with full attention to avoid both premature convergence and divergence.

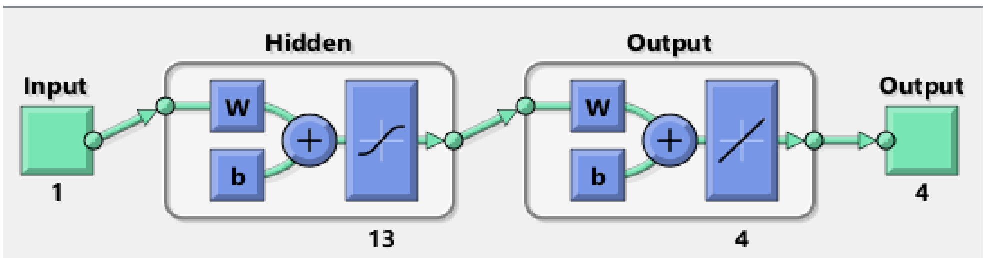


Figure 2: Input/hidden/output layers performances to solve the FO-MML model

4. Numerical measures

This section presents three variations of the FO-MML model based on the values of the FO to obtain the numerical performances.

Case 1: Suppose the FO-MML with $\upsilon = 0.6,\ \lambda_a = 1\ \lambda_b = -1,\ \lambda_c = -1$ and $\lambda_d = -1$ is provided as:

$$\begin{cases} \frac{d^{0.6}}{du^{0.6}} M_r(u) = 1 - M_r(u) - L_i^2(u) + L_r^2(u), & (M_r)_0 = 0.1, \\ \frac{d^{0.6}}{du^{0.6}} M_i(u) = -M_i(u) - 2L_i(u)L_r(u), & (M_i)_0 = 0.2, \\ \frac{d^{0.6}}{du^{0.6}} L_r(u) = -1 - L_r(u) - M_i^2(u) + M_r^2(u), & (L_r)_0 = 0.3, \\ \frac{d^{0.6}}{du^{0.6}} L_i(u) = -L_i(u) + 2M_i(u)M_r(u), & (L_i)_0 = 0.4. \end{cases} \quad \textbf{(4)}$$

Case 2: Consider the FO-MML with $\upsilon = 0.7,\ \lambda_a = 1\ \lambda_b = -1,\ \lambda_c = -1$ and $\lambda_d = -1$ is provided as:

$$\begin{cases} \frac{d^{0.7}}{du^{0.7}} M_r(u) = 1 - M_r(u) - L_i^2(u) + L_r^2(u), & (M_r)_0 = 0.1, \\ \frac{d^{0.7}}{du^{0.7}} M_i(u) = -M_i(u) - 2L_i(u)L_r(u), & (M_i)_0 = 0.2, \\ \frac{d^{0.7}}{du^{0.7}} L_r(u) = -1 - L_r(u) - M_i^2(u) + M_r^2(u), & (L_r)_0 = 0.3, \\ \frac{d^{0.7}}{du^{0.7}} L_i(u) = -L_i(u) + 2M_i(u)M_r(u), & (L_i)_0 = 0.4, \end{cases} \quad (5)$$

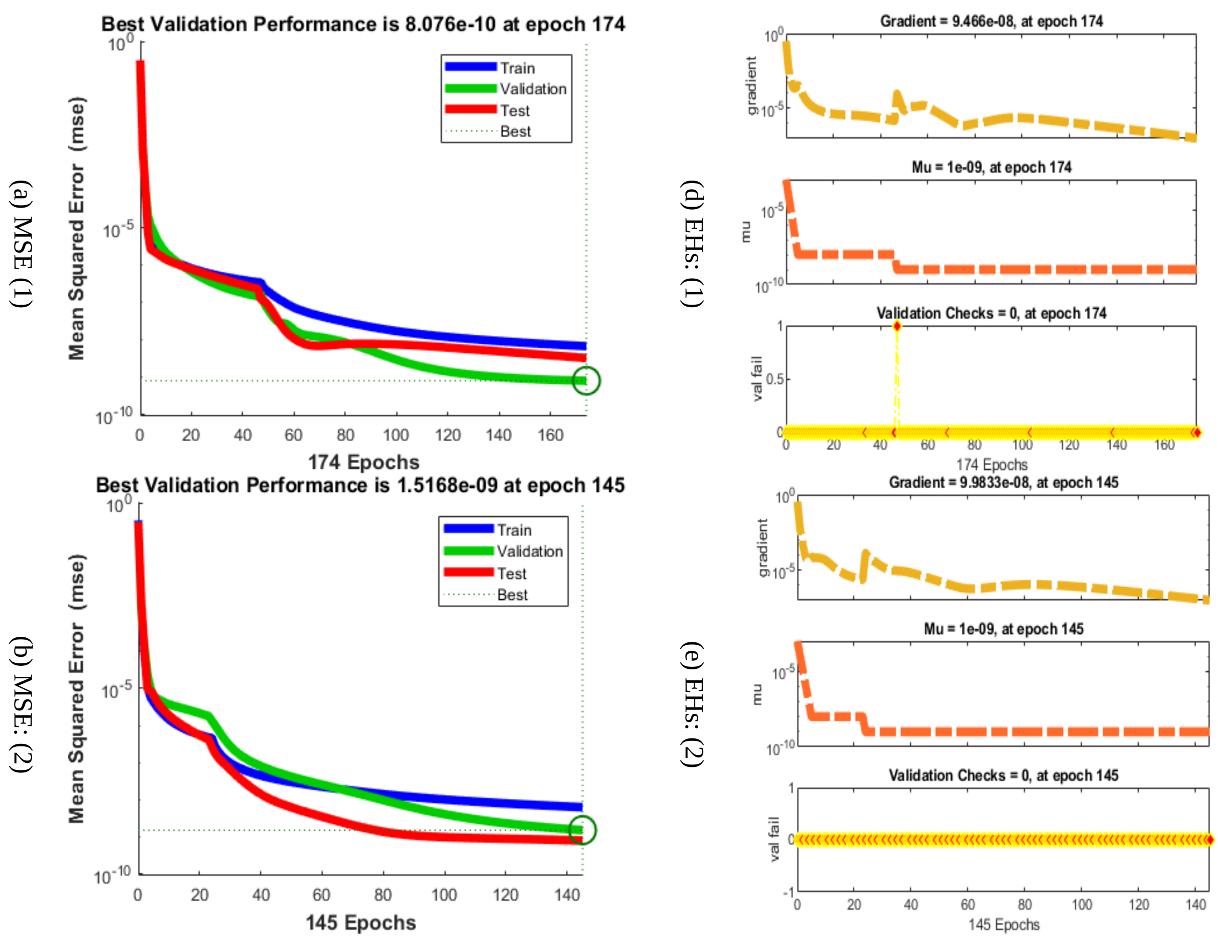
Case 3: Let FO-MML with $\upsilon = 0.8$, $\lambda_a = 1$ $\lambda_b = -1$, $\lambda_c = -1$ and $\lambda_d = -1$ is provided as:

$$\begin{cases} \frac{d^{0.8}}{du^{0.8}} M_r(u) = 1 - M_r(u) - L_i^2(u) + L_r^2(u), & (M_r)_0 = 0.1, \\ \frac{d^{0.8}}{du^{0.8}} M_i(u) = -M_i(u) - 2L_i(u)L_r(u), & (M_i)_0 = 0.2, \\ \frac{d^{0.8}}{du^{0.8}} L_r(u) = -1 - L_r(u) - M_i^2(u) + M_r^2(u), & (L_r)_0 = 0.3, \\ \frac{d^{0.8}}{du^{0.8}} L_i(u) = -L_i(u) + 2M_i(u)M_r(u), & (L_i)_0 = 0.4. \end{cases} \quad (6)$$

The numerical values are illustrated in Figs 3 to 7 for solving the FO-MML story model by using the stochastic framework based on SCGNNs. The obtained numerical simulations of the FO-MML love story model have been provided in Figs 3-5 through the computational stochastic framework. Fig. 3 indicates the performances of MSE using the FO-MML story model. The achieved best measures of FO-MML story model have been performed as 8.07596 × 10^{-10}, 1.51683 × 10^{-09} and 6.59063 × 10^{-10} with iterations 174, 145, and 137 for case 1 to 3. The gradient measures have been reported as 9.47 × 10^{-08}, 9.98 × 10^{-08}, and 9.99 × 10^{-08} for case 1 to 3. The achieved performances and the values of the EHs to solve the FO-MML love story model are presented in Fig 4. The values of the EHs for each case of the FO-MML model have been calculated as 2.85 ×10^{-05}, 4.18 × 10^{-05}, and 5.39 × 10^{-05}. The optimal performances of testing, justification and training have plotted in Fig. 4. The correlation representations are illustrated in Fig. 5 through the SCGNNs procedures to solve the FO-MML love story model. It is observed that each variation of the FO-MML love story model shows the performances of the correlation is 1, which represent the case of perfect system. The accurateness of stochastic SCGNNs for the FO-MML love story model is presented by using the testing, justification, and training procedures. Table 2 designates the MSE to solve the FO-MML love story model using the stochastic SCGNNs.

Table 2: SCGNNs performances for the FO-MML romantic model

Case	MSE			Mu	Performance	Iterations	Gradient	Time
	Training	Testing	Justification					
1	6.80×10^{-09}	8.07×10^{-10}	3.28×10^{-09}	1×10^{-09}	6.80×10^{-09}	175	9.99×10^{-08}	04
2	6.29×10^{-09}	1.51×10^{-09}	7.93×10^{-10}	1×10^{-09}	6.29×10^{-09}	145	7.60×10^{-07}	04
3	3.75×10^{-09}	6.59×10^{-10}	7.67×10^{-09}	1×10^{-09}	3.75×10^{-09}	137	2.69×10^{-06}	03



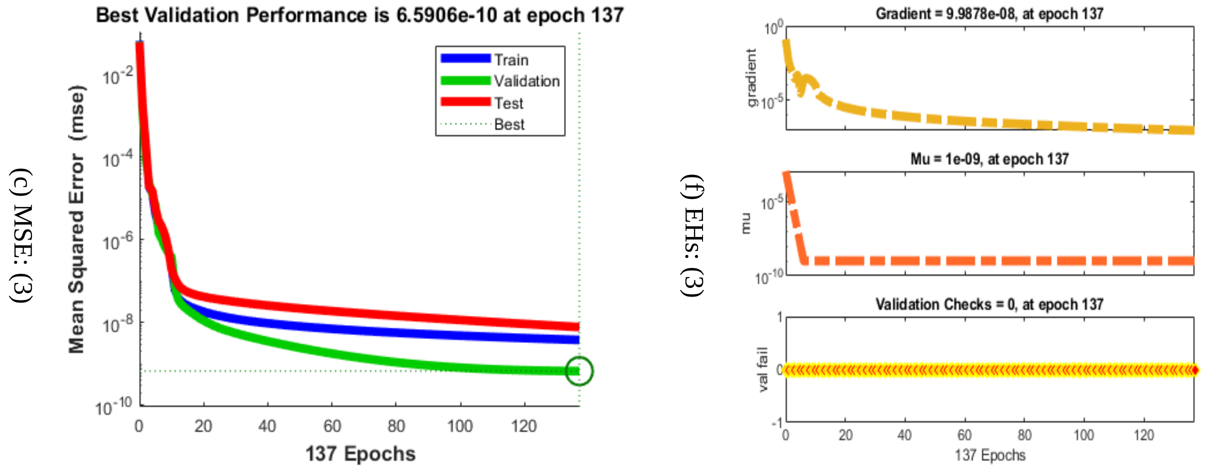
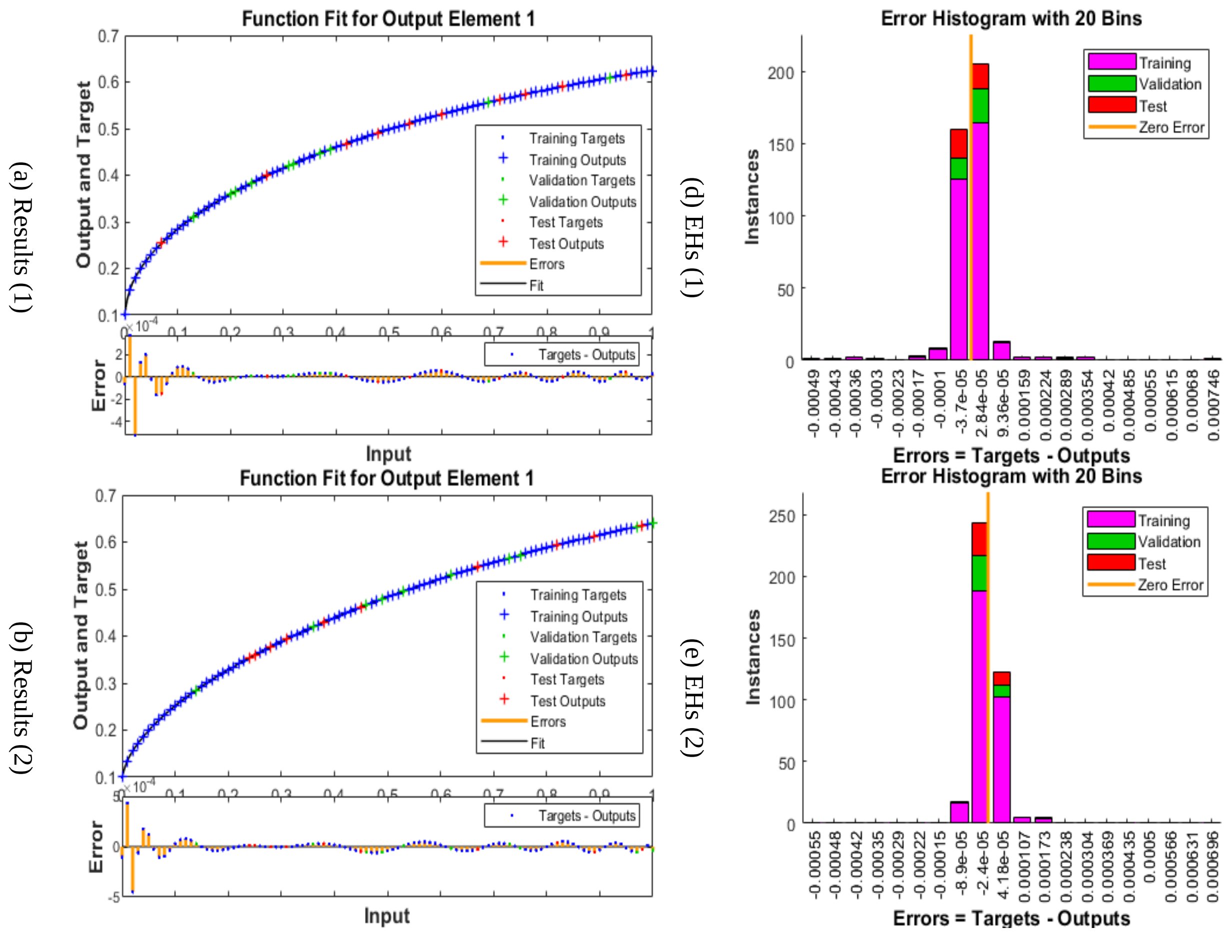


Figure 3: MSE and STs for the FO-MML love story model



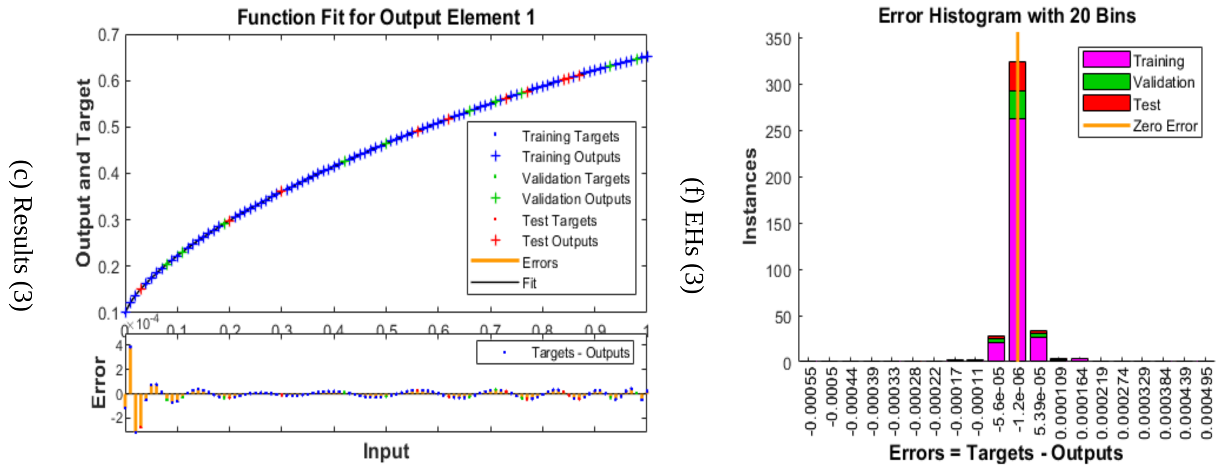
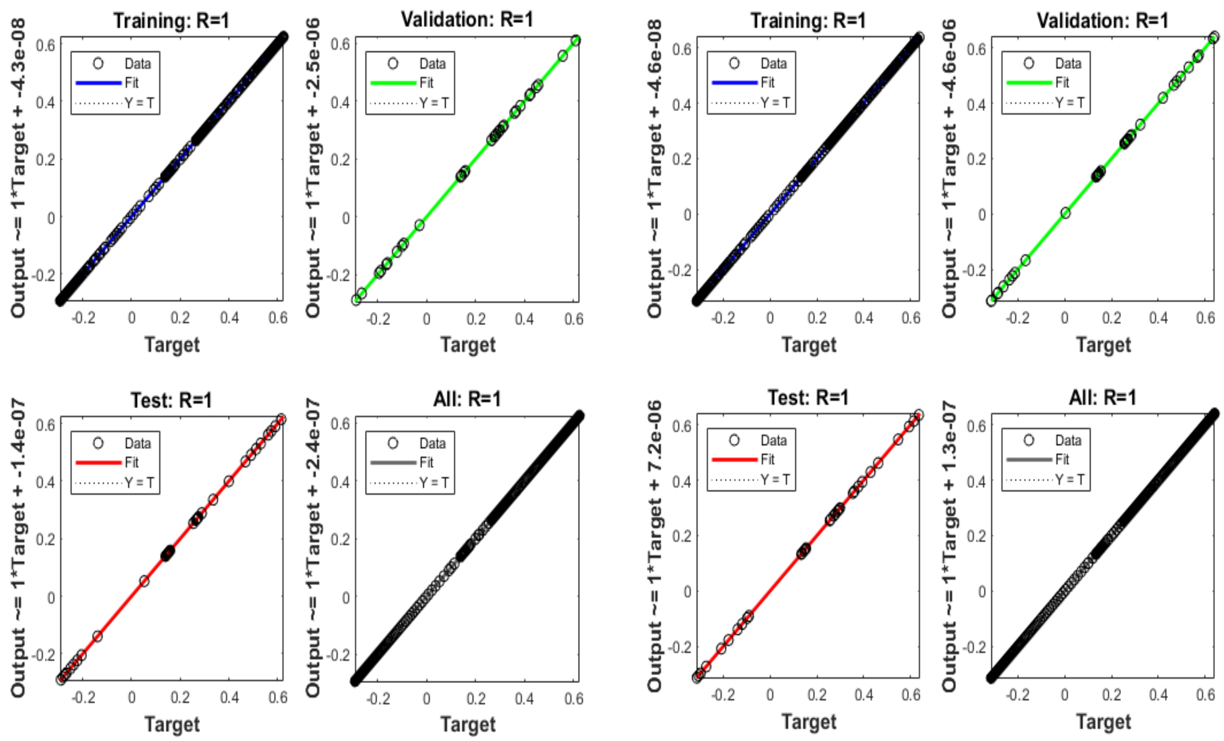
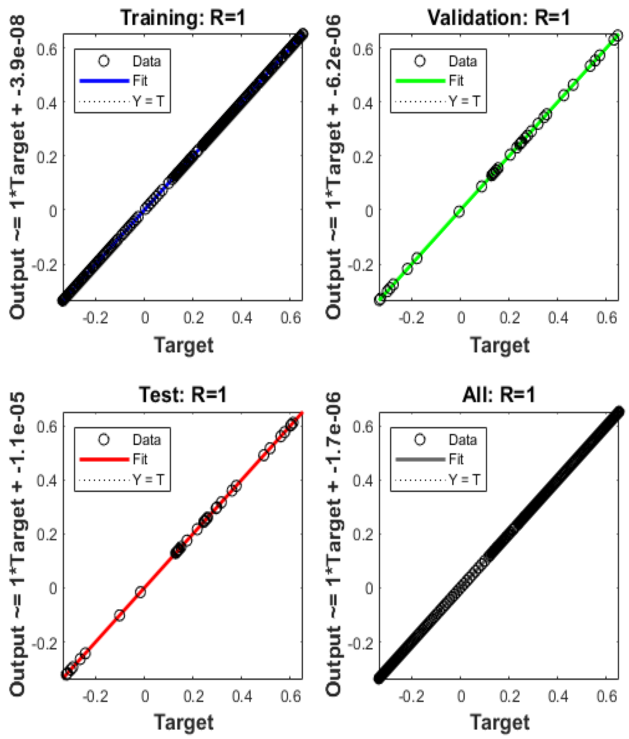


Figure 4: Assessments of the outcomes along with the EHs to solve the FO-MML love story model



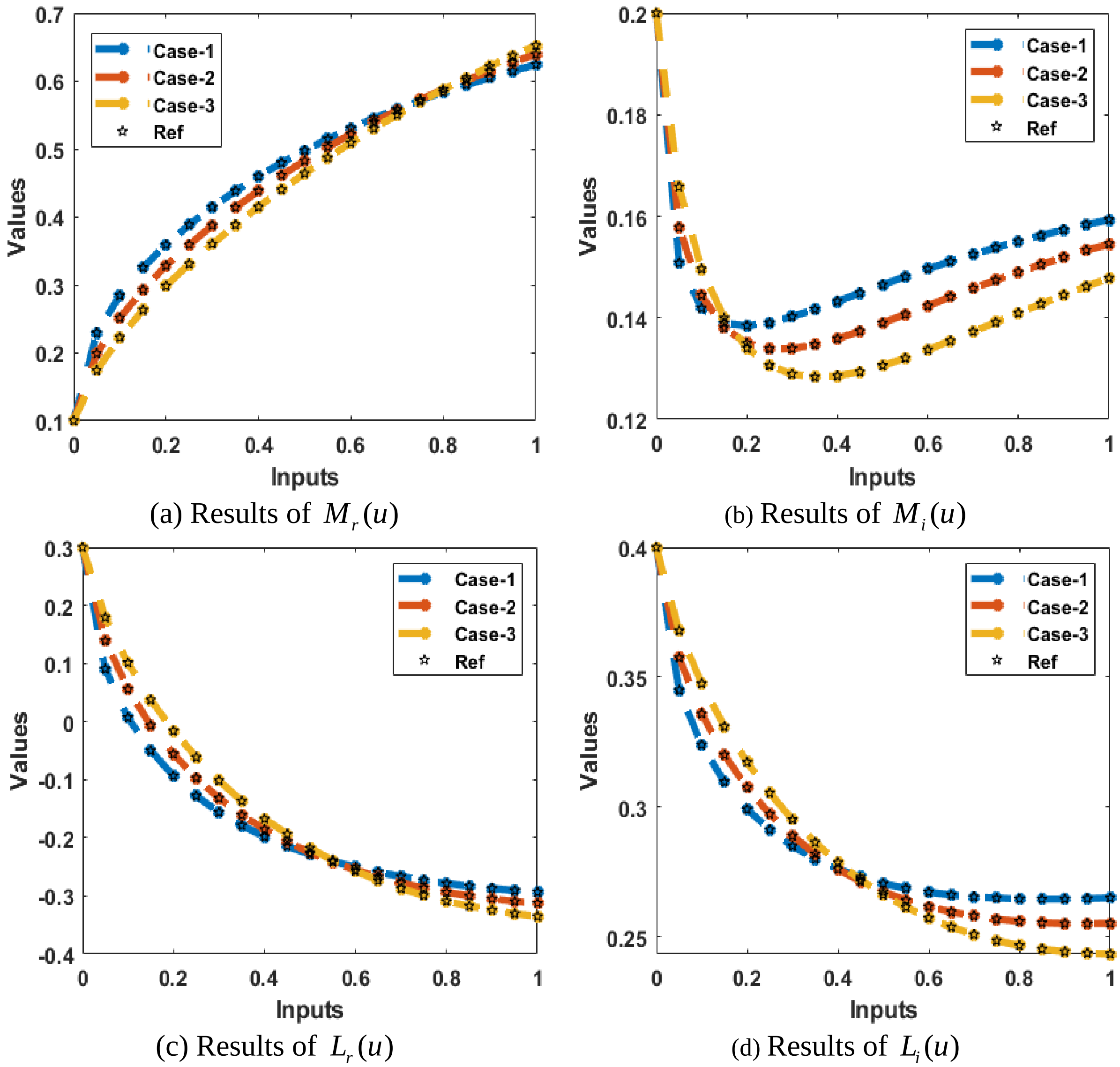
(a) Regression (**1**)   (b) Regression (2**)**



(c) Regression (3)

Figure 5: Regression values to solve the FO-MML love story model

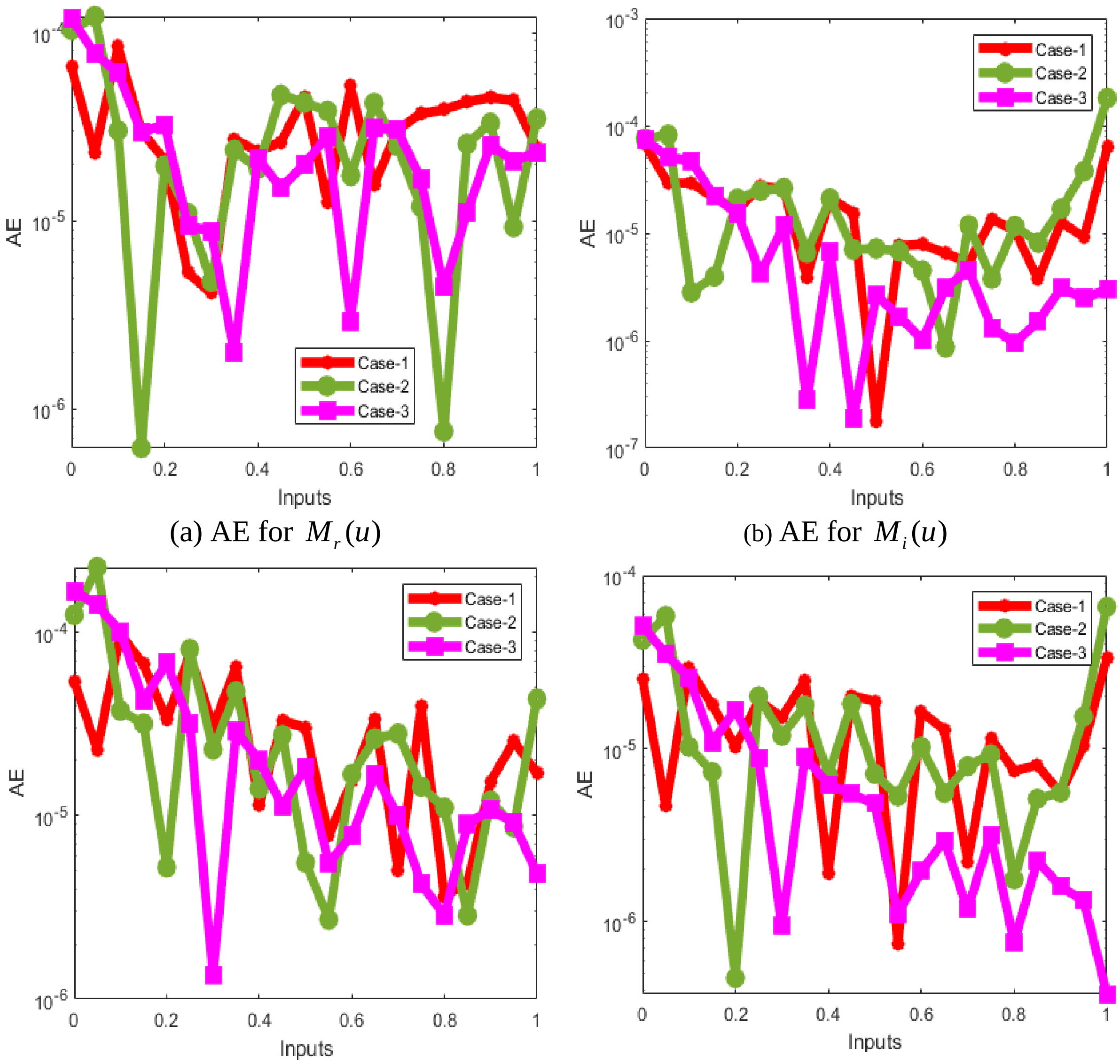
The comparison of the results and AE to solve FO-MML love story model using the stochastic computing SCGNNs scheme has been presented in Figs 6 and 7. The solutions of each dynamic of the FO-MML model are shown in Figs 6. The accuracy of SCGNNs is approved via matching of the proposed and reference solutions. The values of the AE for the dynamics $M_r(u)$, $M_i(u)$, $L_r(u)$ and $L_i(u)$ of the FO-MML love story model are presented in Fig. 7. The AE values for $M_r(u)$ are shown in Fig 7(a), which found around 10^{-04} to 10^{-06}, 10^{-04} to 10^{-07} and 10^{-05} to 10^{-06} for the variations 1 to 3. The AE measures for the dynamics $M_i(u)$ are presented in Fig 7(b), which lie around 10^{-04} to 10^{-06}, 10^{-05} to 10^{-06} and 10^{-05} to 10^{-07} for case 1 to 3. The AE performances for the dynamics $L_r(u)$ and $L_i(u)$ are presented in Fig 7(c), which are found around 10^{-04} to 10^{-05}, 10^{-04} to 10^{-06} and 10^{-05} to 10^{-06} for case 1-3. The matching of the results and small AE values indicate the correctness and precision of the stochastic SCGNNs approach to solve the FO-MML love story model.



(a) Results of $M_r(u)$ (b) Results of $M_i(u)$

(c) Results of $L_r(u)$ (d) Results of $L_i(u)$

Figure 6: Comparison of the achieved performances along with the reference solutions to solve the FO-MML love story model



(a) AE for $M_r(u)$
(b) AE for $M_i(u)$
(c) AE for $L_r(u)$
(d) AE for $L_i(u)$

Figure 7: AE performances to solve the FO-MML love story model using the SCGNNs

6. Conclusion

The current study is related to present the fractional order mathematical model based on the Majnun and Layla romantic story by applying the SCGNNs. The fractional kind of derivatives has been implemented to achieve more accurate and realistic results of the Majnun-Layla romantic story model. The exactness of the model is authenticated by using the comparison of obtained and reference results. The parameter values have been adjusted to find the numerical solutions of the model. Thirteen numbers of neurons have been taken using the statics of the authorization, training, and testing, which have been used as 12%, 77% and 11% for solving the dynamics of the FO-MML model. To prove the dependability, accuracy, and capability of the stochastic SCGNNs scheme, the regression measures, EHs performances, correlation presentations and STs standards have been provided to approve the reliability and consistency. Moreover, the small values of the absolute error enhance the exactness of the proposed scheme.

In future, this computational numerical scheme SCGNNs can be used for stiff nature systems, fluid dynamical systems, biological models [40-50].

Conflict of Interest: There is no conflict of interest between the authors.

References

[1] Baghdadi, G., Jafari, S., Sprott, J.C., Towhidkhah, F. and Golpayegani, M.H., 2015. A chaotic model of sustaining attention problem in attention deficit disorder. Communications in Nonlinear Science and Numerical Simulation, 20(1), pp.174-185.

[2] Sprott, J.C., 2005. Dynamical models of happiness. Nonlinear Dynamics, Psychology, and Life Sciences, 9(1), pp.23-36.

[3] Jafari, S., Ansari, Z., Golpayegani, S.M.R.H. and Gharibzadeh, S., 2013. Is attention a "period window" in the chaotic brain? The Journal of neuropsychiatry and clinical neurosciences, 25(1), pp. E05-E05.

[4] Tabatabaei, S.S., Yazdanpanah, M.J., Jafari, S. and Sprott, J.C., 2014. Extensions in dynamic models of happiness: effect of memory. International Journal of Happiness and Development, 1(4), pp.344-356.

[5] Liao, X. and Ran, J., 2007. Hopf bifurcation in love dynamical models with nonlinear couples and time delays. Chaos, Solitons & Fractals, 31(4), pp.853-865.

[6] Dercole, F. and Rinaldi, S., 2014. Love stories can be unpredictable: Jules et Jim in the vortex of life. Chaos: An Interdisciplinary Journal of Nonlinear Science, 24(2), p.023134.

[7] Breitenecker, F., Judex, F., Popper, N., Breitenecker, K., Mathe, A. and Mathe, A., 2008. Love emotions between laura and petrarch–an approach by mathematics and system dynamics. Journal of computing and information technology, 16(4), pp.255-269.

[8] Newell, A. and Moloney, J., 1992. Nonlinear Optics Addison-Wesley. Reading Mass.

[9] Rozhansky, V.A. and Tsendin, L.D., 2001. Transport phenomena in partially ionized plasma. CRC Press.

[10] Alves-Pires, R., 1996. Nonlinear dynamics in particle accelerators (Vol. 23). World Scientific.

[11] Cveticanin, L., 1995. Resonant vibrations of nonlinear rotors. Mechanism and machine theory, 30(4), pp.581-588.

[12] Mahmoud, G.M. and Aly, S.A., 2000. On periodic solutions of parametrically excited complex non-linear dynamical systems. Physica A: Statistical Mechanics and its Applications, 278(3-4), pp.390-404.

[13] Cveticanin, L., 1992. Approximate analytical solutions to a class of non-linear equations with complex functions. Journal of sound and vibration, 157(2), pp.289-302.

[14] Wu, X., Xu, Y. and Zhang, H., 2011. Random impacts of a complex damped system. International Journal of Non-Linear Mechanics, 46(5), pp.800-806.

[15] Xu,Y., Xu,W., Mahmoud, G.M.:On a complex beam–beam interaction model with random forcing. Phys. A Stat.Mech. Appl. 336(3), 347–360 (2004).

[16] Ghanbari, B. et al., 2020. Mathematical and numerical analysis of a three-species predator-prey model with herd behavior and time fractional-order derivative. Mathematical Methods in the Applied Sciences, 43(4), pp.1736-1752.

[17] Ghanbari, B. et al., 2020. Mathematical analysis of a fractional-order predator-prey model with prey social behavior and infection developed in predator population. Chaos, Solitons & Fractals, 138, p.109960.

[18] Sabir, Z., et al., 2021. An Efficient Stochastic Numerical Computing Framework for the Nonlinear Higher Order Singular Models. Fractal and Fractional, 5(4), p.176.

[19] Sabir, Z., et al., 2022. Numerical investigations of the nonlinear smoke model using the Gudermannian neural networks. Mathematical Biosciences and Engineering, 19(1), pp.351-370.

[20] Sabir, Z., et al., 2021. Design of Morlet wavelet neural network for solving the higher order singular nonlinear differential equations. Alexandria Engineering Journal, 60(6), pp.5935-5947.

[21] Umar, M., et al., 2020. A stochastic intelligent computing with neuro-evolution heuristics for nonlinear SITR system of novel COVID-19 dynamics. Symmetry, 12(10), p.1628.

[22] Umar, M., et al., 2021. Neuro-swarm intelligent computing paradigm for nonlinear HIV infection model with CD4+ T-cells. Mathematics and Computers in Simulation, 188, pp.241-253.

[23] Umar, M., et al., 2021. Integrated neuro-swarm heuristic with interior-point for nonlinear SITR model for dynamics of novel COVID-19. Alexandria Engineering Journal, 60(3), pp.2811-2824.

[24] Sabir, Z., 2022. Stochastic numerical investigations for nonlinear three-species food chain system. International Journal of Biomathematics, 15(04), p.2250005.

[25] Saeed, T., et al., 2022. An advanced heuristic approach for a nonlinear mathematical based medical smoking model. Results in Physics, 32, p.105137.

[26] Umar, M., et al., 2021. A novel study of Morlet neural networks to solve the nonlinear HIV infection system of latently infected cells. Results in Physics, 25, p.104235.

[27] Guirao, J.L., et al., 2020. Design and numerical solutions of a novel third-order nonlinear Emden–Fowler delay differential model. Mathematical Problems in Engineering, 2020.

[28] Sabir, Z., et al., 2020. Integrated intelligent computing with neuro-swarming solver for multi-singular fourth-order nonlinear Emden–Fowler equation. Computational and Applied Mathematics, 39(4), pp.1-18.

[29] Kumar, P., Erturk, V.S. and Murillo-Arcila, M., 2021. A complex fractional mathematical modeling for the love story of Layla and Majnun. Chaos, Solitons & Fractals, 150, p.111091.

[30] Jafari, S., Sprott, J.C. and Golpayegani, S.M.R.H., 2016. Layla and Majnun: a complex love story. Nonlinear Dynamics, 83(1), pp.615-622.

[31] A. Yokuş and S. Gülbahar, Numerical solutions with linearization techniques of the fractional Harry Dym equation. Applied Mathematics and Nonlinear Sciences, 4 (1) (2019), 35-42.

[32] E. İlhan and İ. O. Kıymaz, A generalization of truncated M-fractional derivative and applications to fractional differential equations. Applied Mathematics and Nonlinear Sciences, 5 (1) (2020) 171-188.

[33] S. Momani, R.W. Ibrahim, On a fractional integral equation of periodic functions involving Weyl–Riesz operator in Banach algebras. Journal of Mathematical Analysis and Applications, 339 (2) (2008) 1210-1219.

[34] R. W. Ibrahim and S. Momani, On the existence and uniqueness of solutions of a class of fractional differential equations. Journal of Mathematical Analysis and Applications, 334 (1) (2007), 1-10.

[35] F. Yu, Integrable coupling system of fractional soliton equation hierarchy. Physics Letters A, 373 (41) (2009) 3730-3733.

[36] B. Bonilla, M. Rivero, J. J. Trujillo, On systems of linear fractional differential equations with constant coefficients. Applied Mathematics and Computation, 187 (1) (2007) 68-78.

[37] K. Diethelm, N. J. Ford, Analysis of fractional differential equations. Journal of Mathematical Analysis and Applications, 265 (2) (2002) 229-248.

[38] Huang, L. and Bae, Y., 2018. Chaotic dynamics of the fractional-love model with an external environment. Entropy, 20(1), p.53.

[39] Huang, L. and Bae, Y., 2019. Nonlinear behavior in fractional-order Romeo and Juliet's love model influenced by external force with fuzzy function. International Journal of Fuzzy Systems, 21(2), pp.630-638.

[40] Ilhan, E. et al., 2020. A generalization of truncated M-fractional derivative and applications to fractional differential equations. Applied Mathematics and Nonlinear Sciences, 5(1), pp.171-188.

[41] Baskonus, H.M. et al., 2019. New complex hyperbolic structures to the lonngren-wave equation by using sine-gordon expansion method. Applied Mathematics and Nonlinear Sciences, 4(1), pp.141-150.

[42] Vajravelu, K., et al., 2017. Influence of velocity slip and temperature jump conditions on the peristaltic flow of a Jeffrey fluid in contact with a Newtonian fluid. Applied Mathematics and Nonlinear Sciences, 2(2), pp.429-442.

[43] Selvi, M.S.M. et al., 2019. Application of modified wavelet and homotopy perturbation methods to nonlinear oscillation problems. Applied Mathematics and Nonlinear Sciences, 4(2), pp.351-364.

[44] Gençoğlu, M.T. and Agarwal, P., 2021. Use of quantum differential equations in sonic processes. Applied Mathematics and Nonlinear Sciences, 6(1), pp.21-28.

[45] Baskonus, H.M., Senel, M., Kumar, A., Yel, G., Senel, B. and Gao, W., 2022. On the Wave Properties of the Conformable Generalized Bogoyavlensky–-Konopelchenko Equation. In Handbook of Fractional Calculus for Engineering and Science (pp. 103-119). Chapman and Hall/CRC.

[46] Gao, W., Veeresha, P., Prakasha, D.G. and Baskonus, H.M., 2021. New numerical simulation for fractional Benney–Lin equation arising in falling film problems using two novel techniques. Numerical Methods for Partial Differential Equations, 37(1), pp.210-243.

[47] Palak Barapatre, Yash Ingolikar, Prajakta Desai, Pooja Jajoo, and Prasheel Thakre. (2022). A secured architecture for IoT-based healthcare system. 3C Empresa. Investigación y pensamiento crítico, 11(2), 222-230. https://doi.org/10.17993/3cemp.2022.110250.222-230.

[48] Valentin, Y., Fail, G., y Pavel, U. (2022). Shapley values to explain machine learning models of school student's academic performance during COVID-19. 3C TIC. Cuadernos de desarrollo aplicados a las TIC, 11(2), 136-144. https://doi.org/10.17993/3ctic.2022.112.136-144.

[49] Dharmik, R. C., Chavhan, S., y Sathe, S. R. (2022). Deep learning based missing object detection and person identification: an application for smart CCTV. 3C Tecnología. Glosas de innovación aplicadas a la pyme, 11(2), 51-57. https://doi.org/10.17993/3ctecno.2022.v11n2e42.51-57.

[50] Zouhair D., Maria Teresa B., Miguel Ángel L. and Raquel M.(2022). Limit cycles of perturbed global isochronous center. 3C Tecnología. Glosas de innovación aplicada a la pyme, 11 (2), 25-36. https://doi.org/10.17993/3ctecno.2022.v11n2e42.25-36.