A computational approach to solve the nonlinear biological prey-predator system

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Abstract: This study is to solve a nonlinear biological prey-predator system (NBPPS) using a novel design of Levenberg-Marquardt backpropagation approach (LMBA). The LMBA based supervised neural networks (SNNs) with three kinds of sample data, training, validation and testing. The percentages for these data to solve three different cases of the NBPPS are selected for training 75%, validation 10% and testing 15%, respectively. The numerical performances of the Adams method are used for the reference dataset to solve the NBPPS. The obtained form of the numerical solutions of the NBPPS based on the SNNs along with LMBA is used to reduce the functions of mean square error (MSE). For the correctness, competence, competence and effectiveness of the proposed SNNs along with LMBA, the numerical procedures are proficient based on the proportional schemes and analyses in terms of MSE results, correlation, error histograms and regression.

Keywords: Supervised neural networks; Biological prey-predator system; Levenberg-Marquardt Backpropagation; Reference dataset; Numerical results.

1. Introduction

The nonlinear biological prey-predator system (NBPPS) discovered by Lotka and Volterra before few centuries [1]. The mathematical form of the biological structures is formulated with the system of nonlinear differential equations, e.g., mosquito dispersal model, COVID-19 based SITR system, dengue fever model and SIR model for spreading treatment and infection [2-6]. The mathematical formulation of the NBPPS is obtained using the association of two living animals, rabbits and foxes. It is obvious that the rabbits eat the clover and the foxes eat the rabbits. The rabbits increase when the foxes decrease and the other way around [7]. The decreasing/increasing behavior of these two animals' types shows a NBPPS. The NBPPS has a great importance and implication in the model of mathematical ecology due to its worldwide significance [8]. The generic form of NBPPS is shown as [9]:

$$\begin{cases} f'(x) = f(x)\left(p - qg(x)\right), \\ g'(x) = -g(x)\left(m - nf(x)\right), \\ f(0) = \lambda_1, \qquad g(0) = \lambda_2. \end{cases} \tag{1}$$

Where p, q, m and n are the constant terms, λ_1 and λ_2 represent the initial conditions (ICs). The terms $f(x)$ and $g(x)$ indicate the prey and predator at the time x, respectively. The NBPPS is proposed by the Lotka-Volterra before few centuries and then Holling [10] worked on three different categories of the functional responses. In the applications of biomathematics, many researchers implemented both simulation and modeling to solve a system of nonlinear ordinary differential systems. These models having nonlinearity are usually considered tough, stiff, complex and unrealistic to solve numerically and analytically. Few investigations to solve the NBPPS have been proposed by Elabbasy et al. [11], Liu et al. [12], Danca et al. [13] and Jing et al. [14].

In recent years, various approaches have been proposed to find the approximate results of the NBPPS including Runge–Kutta–Fehlberg approach, differential transformation scheme, method of Laplace Adomian decomposition, Homotopy analysis scheme, Sumudu decomposition technique, finite element approach, fractional reduced differential transformation scheme, confidence domain approach and Adams implicit method [15-21]. All above mentioned schemes have weaknesses, disadvantages and suitability. However, the stochastic procedures based on the proposed SNNs along with LMBA have never been implemented to solve the NBPPS.

Three different cases of the NBPPS have been numerically evaluated using the LMB based SNN approach. The comparison of the results using the LMB based SNN approach and the reference Adams dataset results is also presented. The percentages for this data to solve three different NBPPS cases are selected for training 75%, validation 10% and testing 15%, respectively. The stochastic procedures have been implemented in diverse recent applications [22-28]. Few stochastic solvers applications are nonlinear singular models of the third kind [29-30], SITR system [31-32], Thomas–Fermi equation [33], periodic differential system [34-35], heat conduction model [36], multi-singular systems [37-38], dengue fever system [39], singular prediction, pantograph/delayed form of the systems [40-42], fractional order differential systems [43-44] and functional systems [45-47]. These submissions not only validate the implication of stochastic solvers but also inspire the authors to present a reliable, robust, accurate and stable platform for solving the NBPPS. Hence, the stochastic procedures based on the proposed SNNs along with LMBA have been proposed to solve the NBPPS. Some novel topographies of the current work are described as:

- An integrated computing, novel approach is presented through the designed SNNs along with LMBA to scrutinize the NBPPS.
- The proposed SNNs along with LMBA is available using the Adams method-based dataset for solving different cases of the NBPPS.
- The exactly overlapping of the results with good performances of the absolute error (AE) enhances the worth of the proposed SNNs along with LMBA for solving the NBPPS.
- The presentation through relative investigations on MSE, regression, correlation metrics and error histograms (EHs) authenticate the proposed SNNs along with LMBA.

The other parts of the paper are described as: The numerical solutions of the proposed SNNs along with LMBA to solve the NBPPS are presented in Sect 2. The proposed SNNs along with LMBA to solve the NBPPS with crucial explanations is presented in Sect 3. The concluding remarks with latent associated investigations together with the future research guidance are provided in the last Sect.

2. Methodology

The suggested SNNs along with LMBA is provided in two steps to solve the numerical procedure of the NBPPS.

- Essential interpretations are provided to form the SNNs along with LMBA.
- Execution procedures support the SNNs along with LMBA for solving the NBPPS.

The proposed SNNs along with LMBA is a suitable combination of the multi-layer optimization procedures, which is illustrated in Fig 1. Proposed form of the system for a single neuron is described in Fig 2. The suggested SNNs along with LMBA work with the 'nftool' in the 'Matlab' software is provided using the data based on training 75%, validation 10% and testing 15%, respectively.

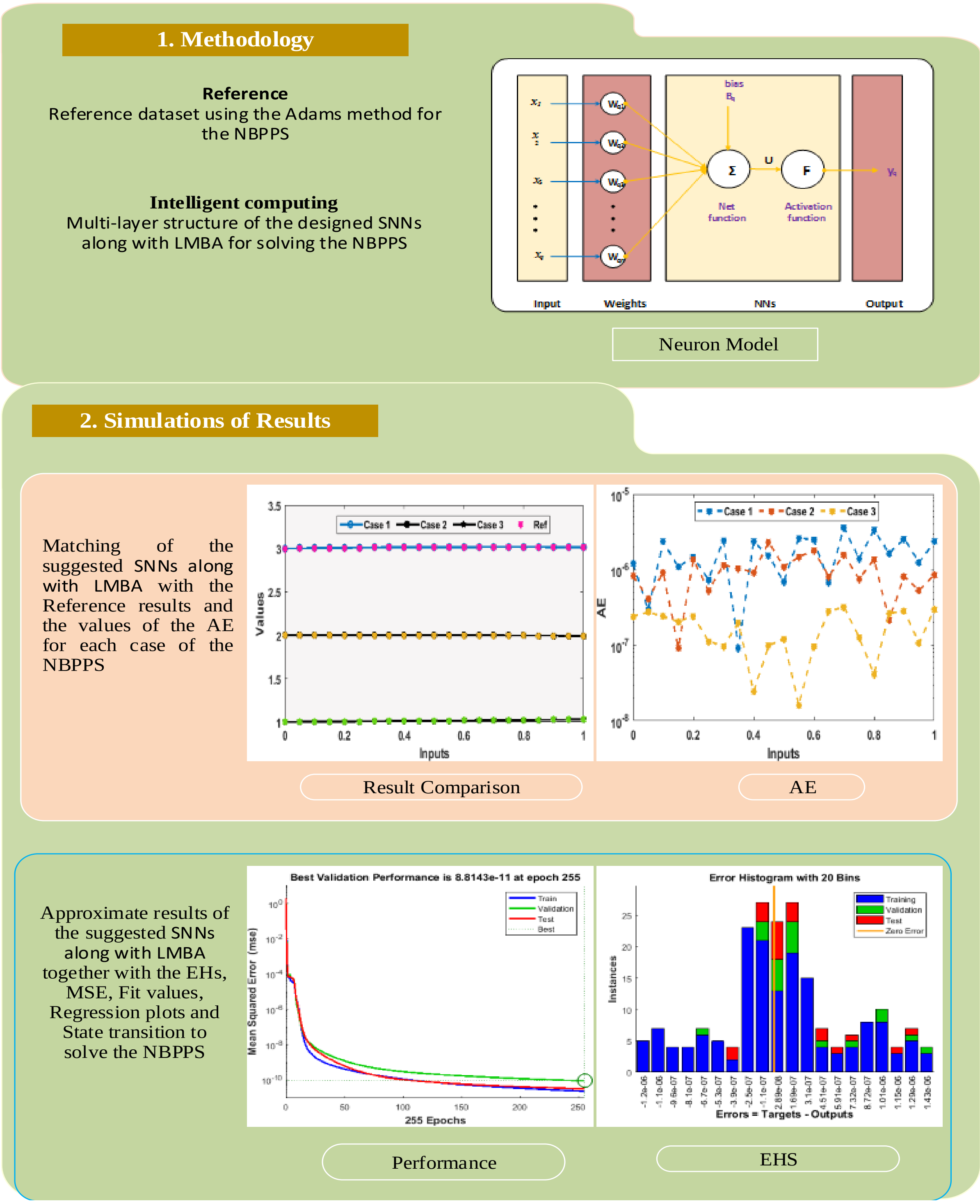


Fig 1: Workflow diagram of the SNNs along with LMBA for solving the NBPPS

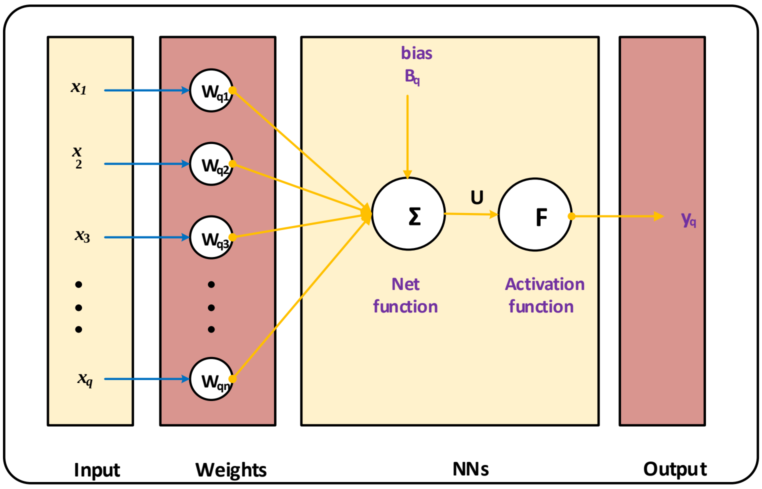


Fig 2: Proposed structure for the single neuron

3. Numerical simulations

In this section, the solution of three different cases of the NBPPS is presented using the SNNs along with LMBA. The mathematical form of these cases is presented as:

Case- 1: Consider a NBPPS with $p=0.1$, $q=0.014$, $m=0.012$, $n=0.08$, $\lambda_1=3$ and $\lambda_2=6$. The mathematical form of the Eq (1) using the above constant values is given as:

$$\begin{cases} f'(x)=0.1f(x)-0.014f(x)g(x), \\ g'(x)=-0.012g(x)+0.08f(x)g(x), \\ f(0)=3, \qquad g(0)=6. \end{cases} \tag{2}$$

Case- 2: Consider a NBPPS with $p=0.1$, $q=0.024$, $m=0.012$, $n=0.08$, $\lambda_1=2$ and $\lambda_2=4$. The mathematical form of the Eq (1) using the above constant values is given as:

$$\begin{cases} f'(x)=0.1f(x)-0.024f(x)g(x), \\ g'(x)=-0.012g(x)+0.08f(x)g(x), \\ f(0)=2, \qquad g(0)=4. \end{cases} \tag{2}$$

Case- 3: Consider a NBPPS with $p=0.1$, $q=0.034$, $m=0.012$, $n=0.08$, $\lambda_1=1$ and $\lambda_2=2$. The mathematical form of the Eq (1) using the above constant values is given as:

$$\begin{cases} f'(x)=0.1f(x)-0.034f(x)g(x), \\ g'(x)=-0.012g(x)+0.08f(x)g(x), \\ f(0)=1, \qquad g(0)=2. \end{cases} \tag{4}$$

The numerical performances are provided using the SNNs along with LMBA for solving the NBPPS in inputs [0, 1] with 0.01 step size. The 'nftool' Matlab command is implemented to solve the NBPPS using the 10 neurons along with the selection of training data 75%, validation data 10% and testing data 15%. The obtained results through SNNs along with LMBA for solving the NBPPS are provided in Fig. 3, however, the SNNs along with LMBA is implemented to assess each type of the NBPPS.

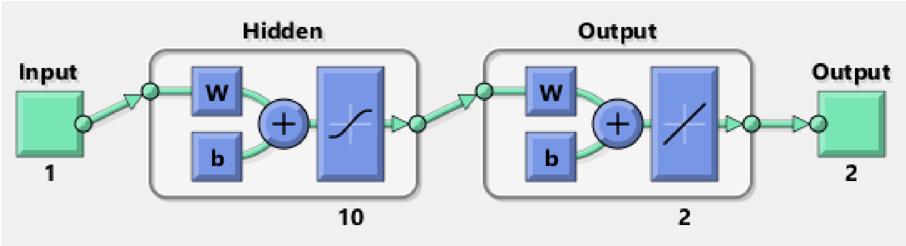
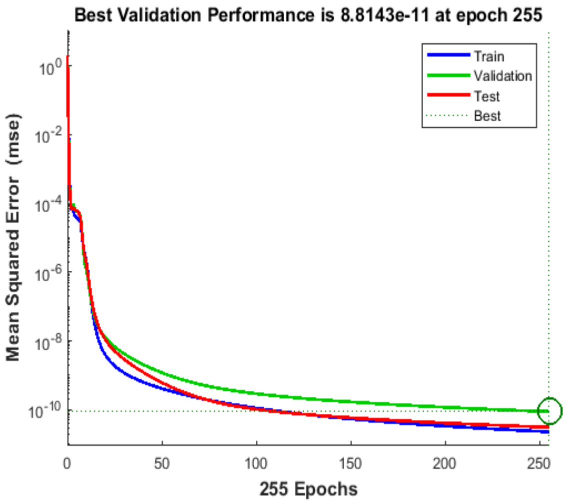


Fig 3: Proposed SNNs along with LMBA for solving the NBPPS

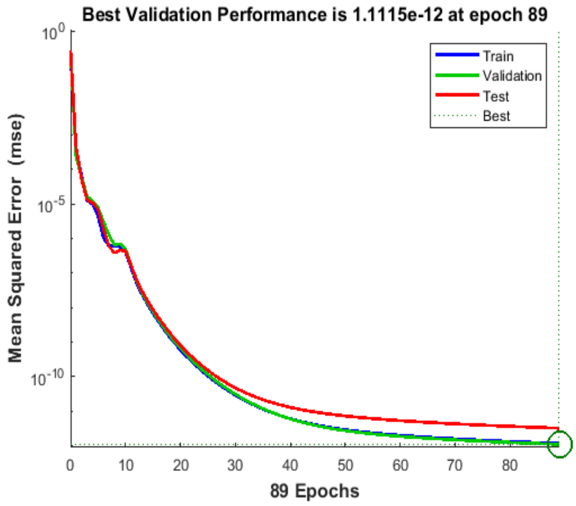
The graphs of the proposed SNNs along with LMBA for solving the NBPPS are illustrated in Figs 4 to 12. The attained numerical performances of each case of the NBPPS using the state of performance as well as transition are illustrated in Figs 4 to 5. The MSE calculated values for the states of training, best curve, validation and testing are available to solve NBPPS are derived in Fig 4. The best performances of the NBPPS are attained at epoch 225, 89 and 38 lie around 2.28×10^{-11}, 1.17×10^{-12} and 4.18×10^{-13}, respectively. Fig 5 illustrates the gradient performances of the SNNs along with LMBA for solving the NBPPS are 9.94×10^{-08}, 9.89×10^{-08} and 9.92×10^{-08}. These graphs represent the precision and accuracy, as well as, convergence of the suggested SNNs along with LMBA for solving the NBPPS. Figs 6-8 indorse the plots of fitting curves for each case of the NBPPS that proves the comparation of the obtained outcomes through the suggested SNNs along with LMBA using the reference dataset. The maximum error plots are drawn using the testing, training and validation through suggested SNNs along with LMBA for solving the NBPPS. The EHs values are provided in Fig. 9, whereas the regression values are drawn in Figs 10 to 12 for solving the NBPPS. The correlation soundings are functional to authenticate the regression analysis. It is observed that the correlation (R) found around 1 for solving the NBPPS, which describes the perfect model. The testing, validation and training graphs indicate the exactness of the suggested SNNs along with LMBA for solving the NBPPS. Moreover, the convergence measures for the MSE are found capable for training, validation, epochs, testing, complexity investigations and backpropagation values are provided in Table 1 to solve the NBPPS.

Table 1: Proposed SNNs along with LMBA for solving the NBPPS

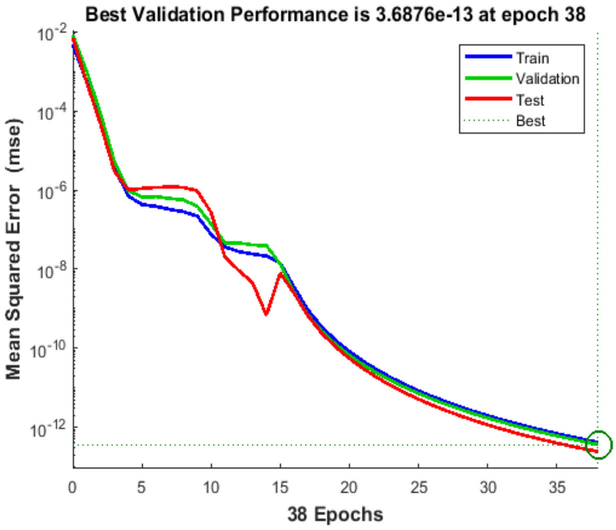
Case	MSE			(Performance)	Gradient	Mu	Epoch	Time
	(Training)	(Validation)	(Testing)					
I	2.28×10^{-11}	8.81×10^{-11}	3.09×10^{-11}	2.28×10^{-11}	9.94×10^{-08}	1.00×10^{-09}	225	1
II	1.17×10^{-12}	1.11×10^{-12}	3.23×10^{-12}	1.17×10^{-12}	9.89×10^{-08}	1.00×10^{-11}	89	1
III	4.17×10^{-13}	3.68×10^{-12}	2.42×10^{-11}	4.18×10^{-13}	9.92×10^{-08}	1.00×10^{-13}	38	1



(a) Case 1: MSE for the NBPPS

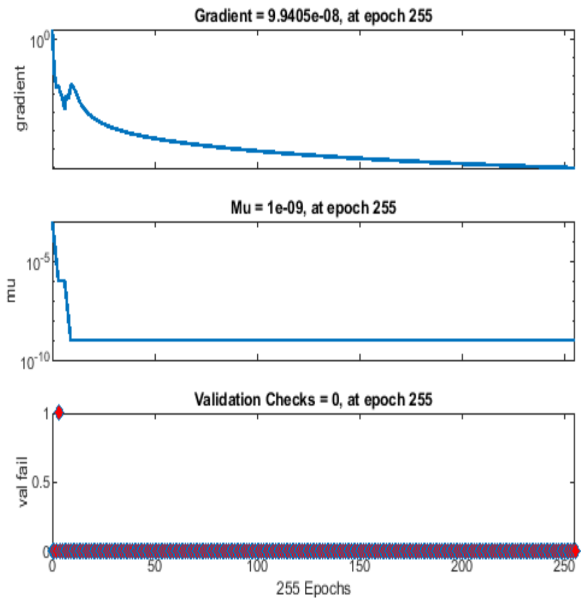


(b) Case 2: MSE for the NBPPS

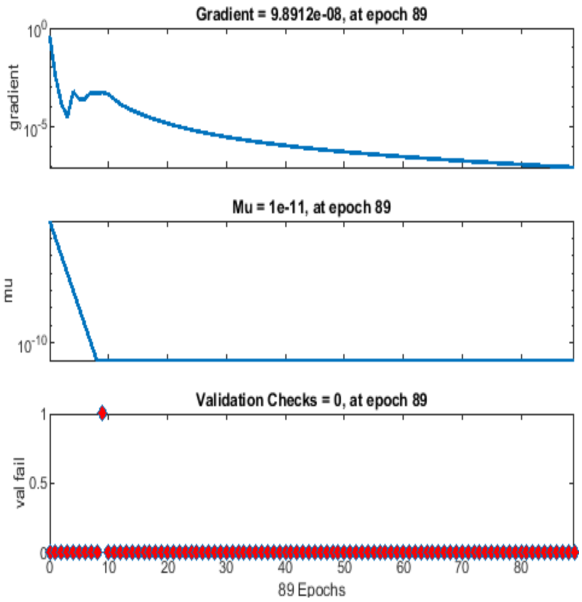


(c) Case 3: MSE for the NBPPS

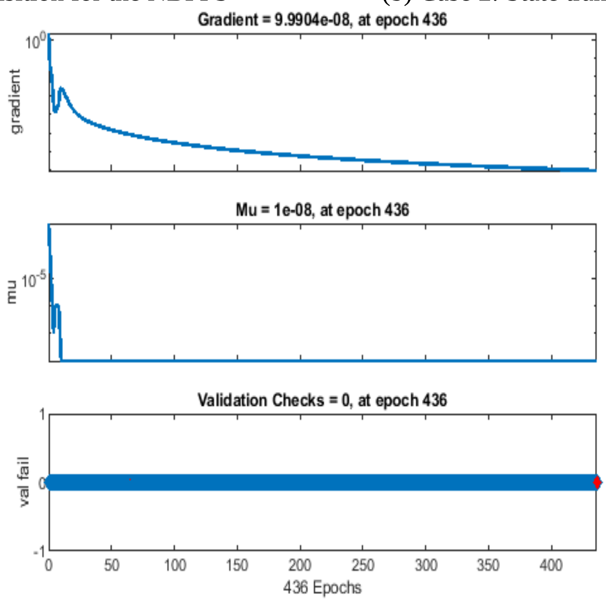
Fig 4: Performance based on the MSE values using the SNNs along with LMBA for solving the NBPPS



(a) Case 1: State transition for the NBPPS



(b) Case 2: State transition for the NBPPS



(c) Case 3: State transition for the NBPPS

Fig 5: State transition values based on the SNNs along with LMBA for solving the NBPPS

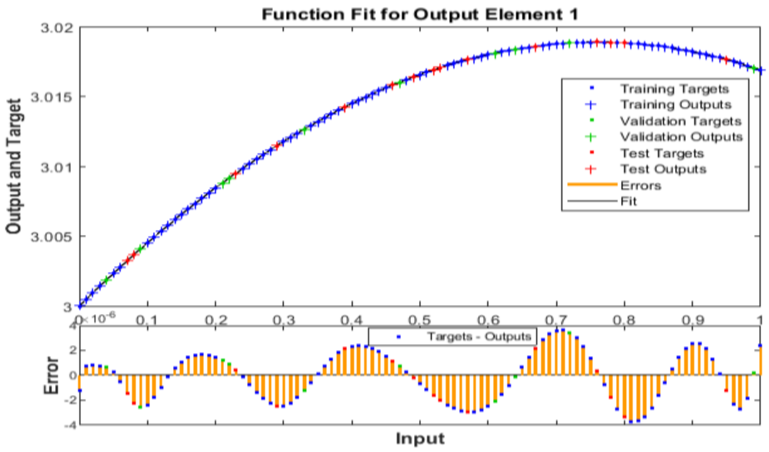


Fig 6: **Case 1:** Result comparison through SNNs along with LMBA for solving the NBPPS

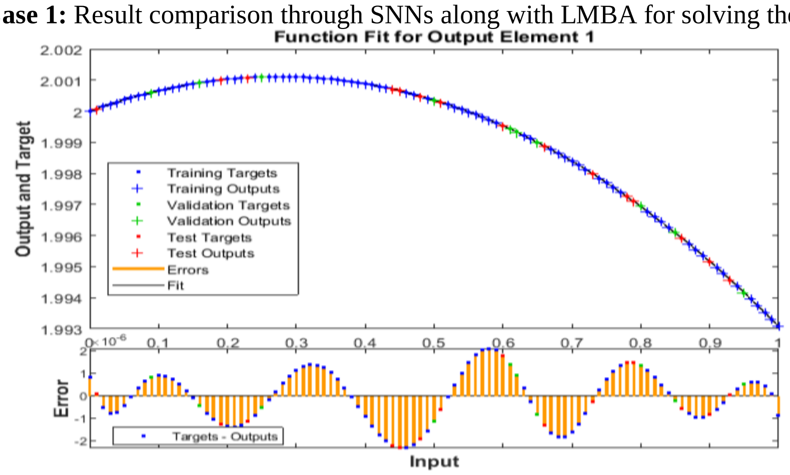


Fig 7: **Case 2:** Result comparison through SNNs along with LMBA for solving the NBPPS

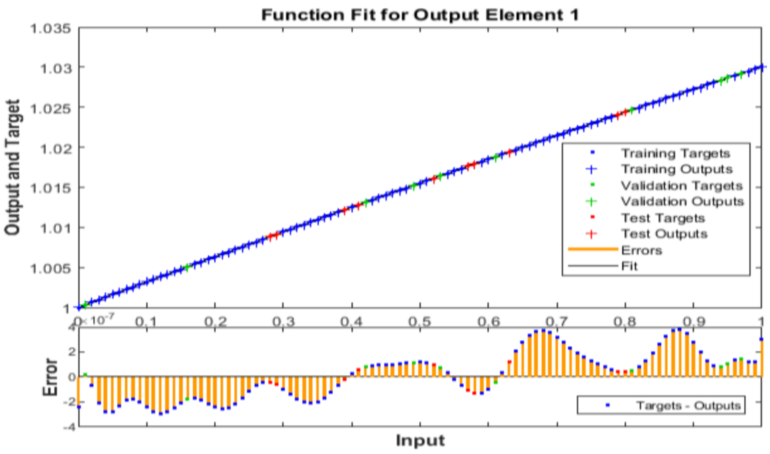
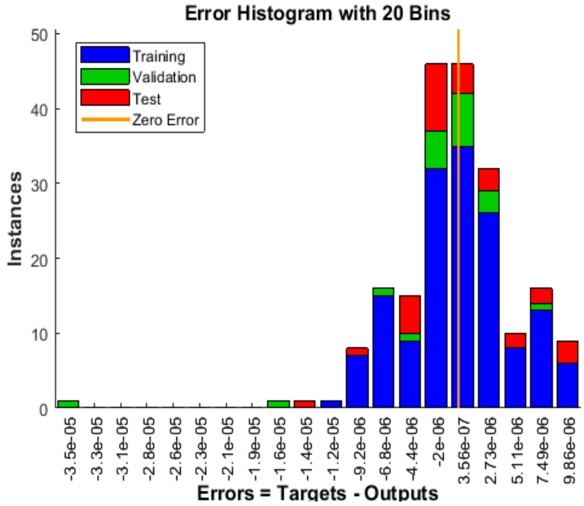
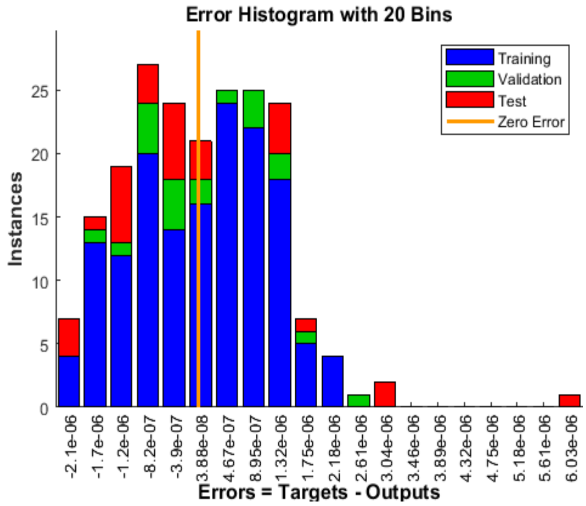


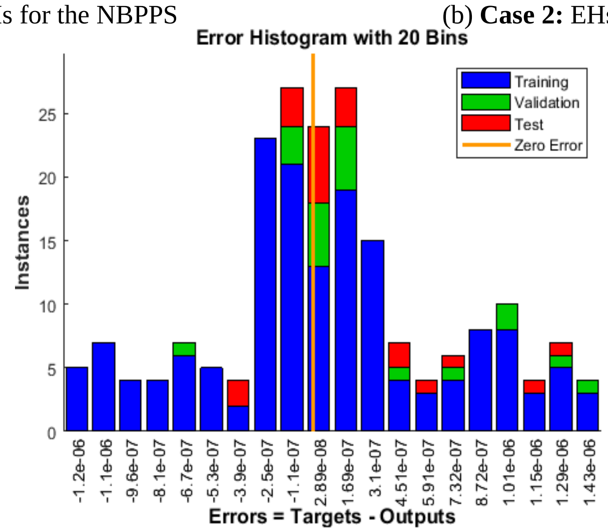
Fig 8: **Case 3:** Result comparison through SNNs along with LMBA for solving the NBPPS



(a) **Case 1:** EHs for the NBPPS



(b) **Case 2:** EHs for the NBPPS



(c) **Case 3:** EHs for the NBPPS

Fig 9: EHs for the through SNNs along with LMBA for solving the NBPPS

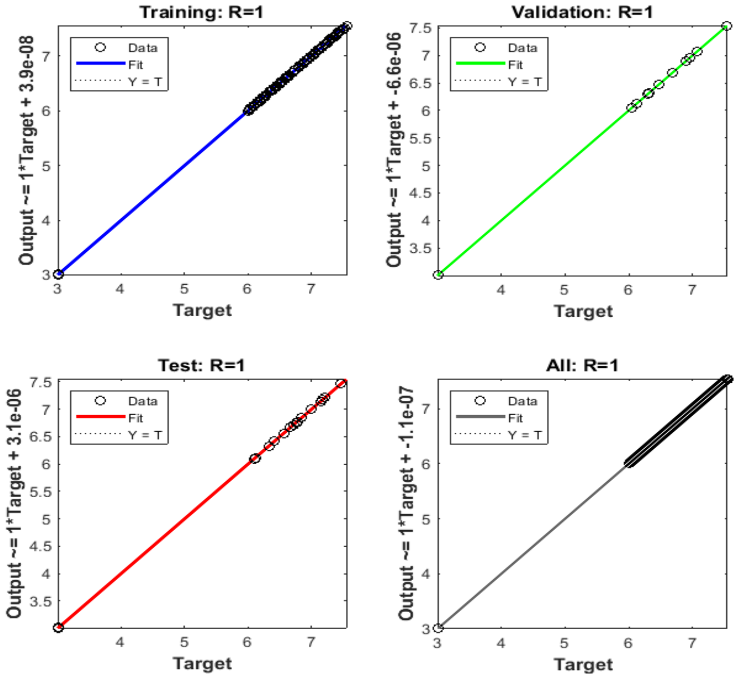


Fig 10: Case 1: Regression plots for the NBPPS

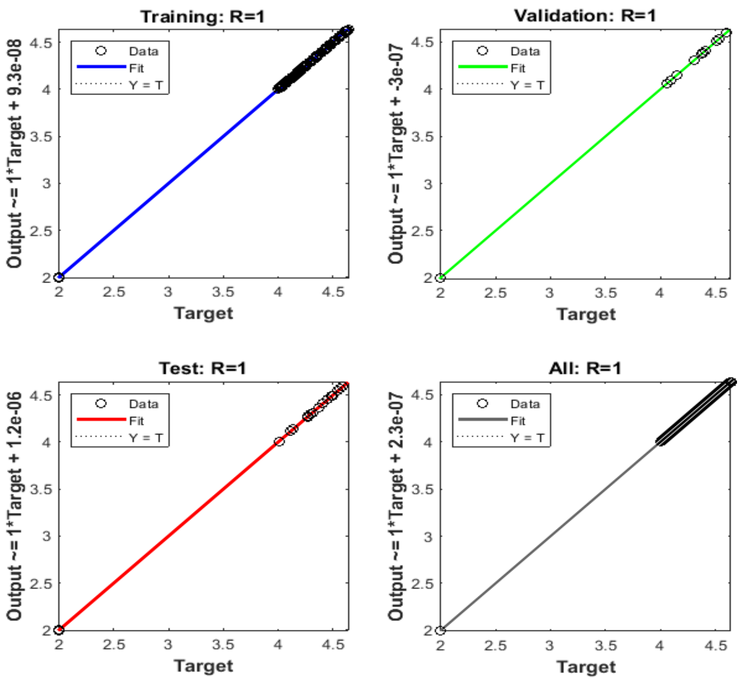


Fig 11: Case 2: Regression plots for the NBPPS

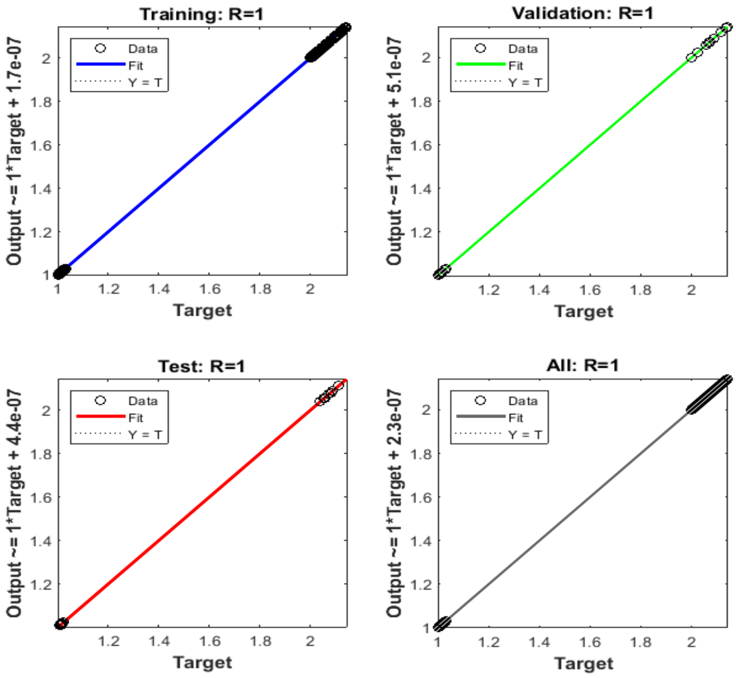
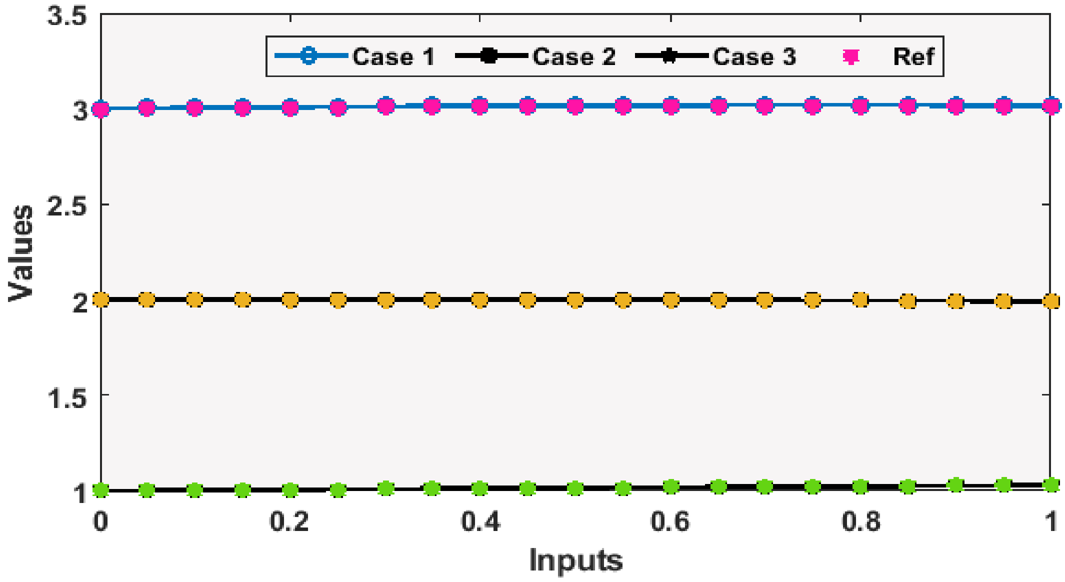
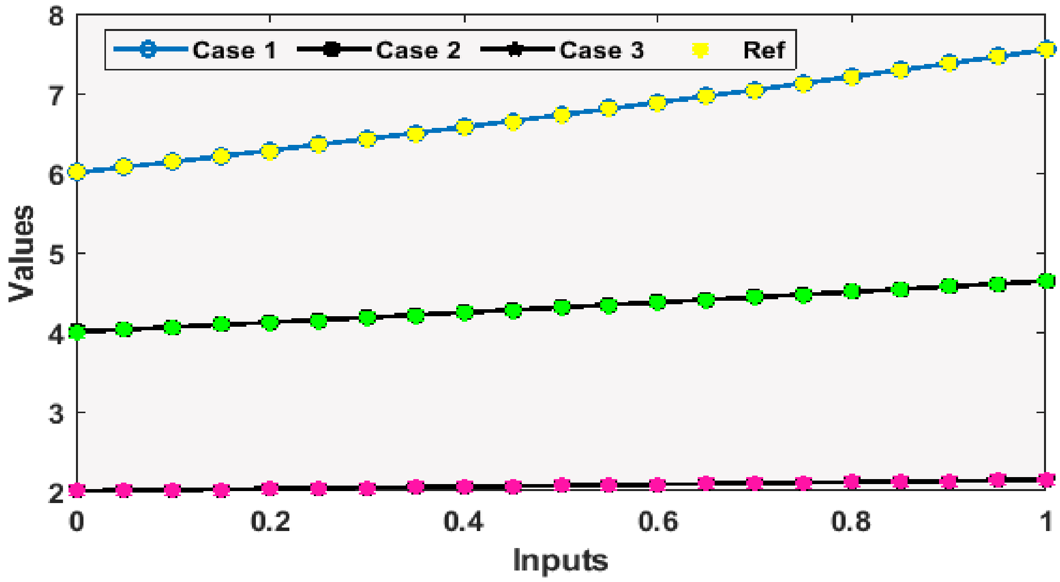


Fig 12: Case 3: Regression plots for the NBPPS

The comparison plots are drawn in Figs 13 and 14 to solve each case of the NBPPS. The results of the parameters $f(x)$ and $g(x)$ using the SNNs along with LMBA are illustrated in the subfigures 13(a) and 13(b). It is seen that the overlapping outcomes indicate the exactness and accuracy of the designed SNNs along with LMBA. The values of the absolute error (AE) are drawn for each case of the NBPPS are demonstrated in Fig 14. The parameters $f(x)$ and $g(x)$ values for each case are illustrated in Fig 14 and the parameters $f(x)$ and $g(x)$ values found around [10^{-06}, 10^{-08}] and [10^{-05}, 10^{-07}]. These overlapped and matching performances of the AE authenticate the correctness of the SNNs along with LMBA.

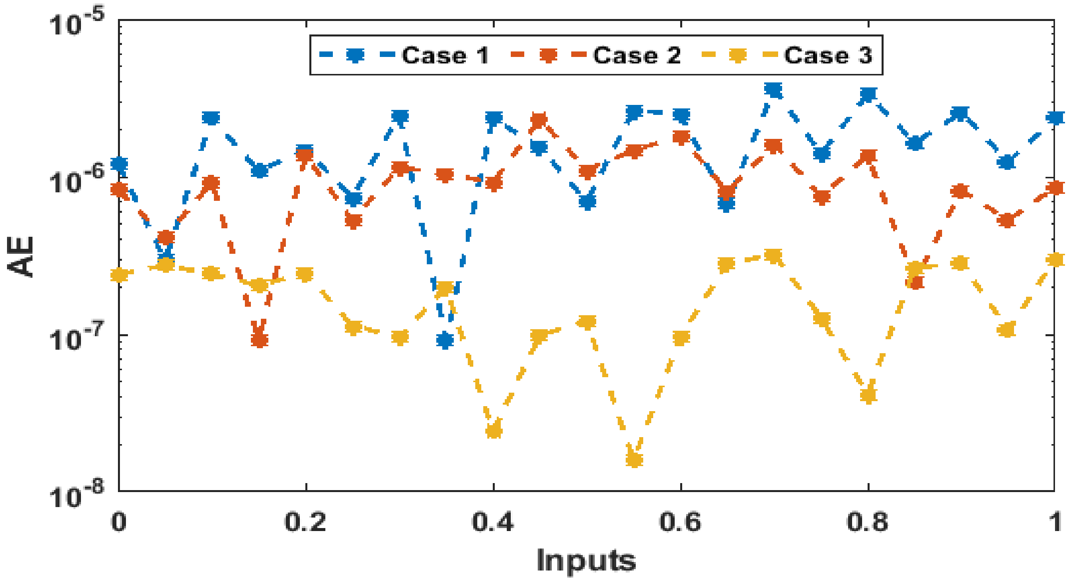


(a) Results of $f(x)$ for cases 1-3

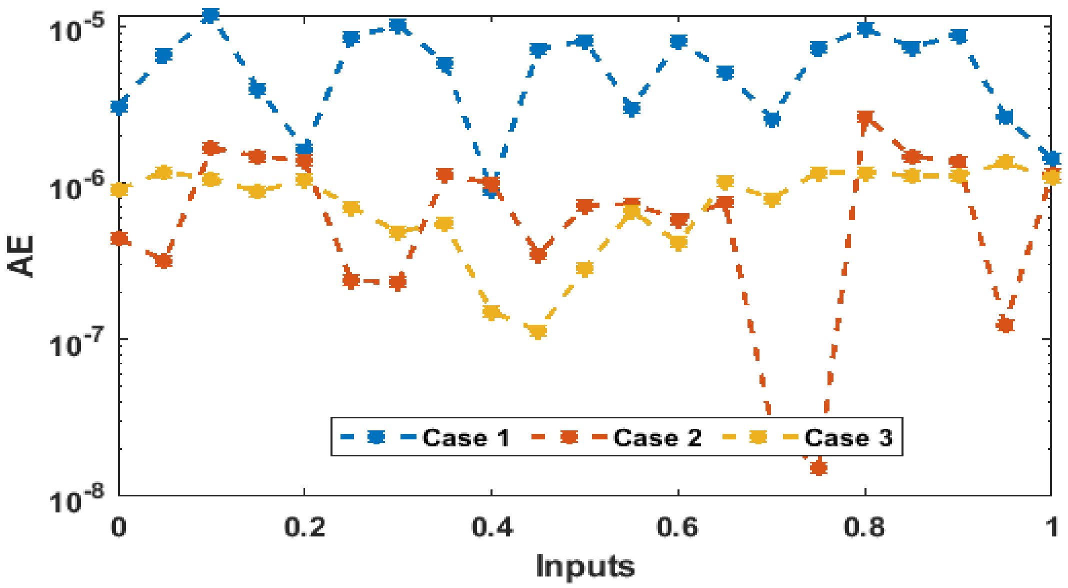


(a) Results of $g(x)$ for cases 1-3

Fig 13: Comparison performance based on the SNNs along with LMBA for solving the NBPPS



(a) Values of the AE for $f(x)$ using cases 1-3



(b) Values of the AE for $g(x)$ using cases 1-33

Fig 14: AE values for the indexes prey and predator to solve the NBPPS using the SNNs along with LMBA

5. Conclusions

The purpose of the current work is to handle numerically the nonlinear biological prey-predator system (NBPPS) using the supervised neural networks (SNNs) along with the Levenberg-Marquardt backpropagation approach (LMBA). The NBPPS has a great history and treated numerically by using various schemes. The SNNs along with LMBA is applied with three kinds of sample data training, validation and testing. The percentages for these data to solve three different cases of the NBPPS are selected for training 75%, validation 10% and testing 15%, respectively. For the brilliance, quality, perfection and exactness of the SNNs along with LMBA, the overlapping of the accomplished results is found to solve each case of the NBPPS. The performances of the MSE and convergence are applied to the testing, validation, training and best curve for each of the NBPPS. The correlation presentations are accomplished to validate the regression measures. The values of the gradient using the step size are achieved in each case of the NBPPS. Furthermore, the precision, exactness, correctness is resolute using the graphical and numerical configurations of the EHs, convergence plots, the MSE catalogues and regression dynamics, respectively.

In future, the proposed SNNs along with LMBA can be exploited and explored solve the fractional order problems, lonngren-wave equation and fluid models [48-54].

Conflict of interest: All the authors state that there are no potential conflicts of interest.

Funding: This research work was funded by Institutional Fund Projects under grant no. (IFPIP: 82-130-1442). Therefore, authors gratefully acknowledge technical and financial support from Ministry of Education and King Abdulaziz University, DSR, Jeddah, Saudi Arabia.

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